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ON NUMERICAL MODELING OF RESERVOIR GEOMECHANICAL PROBLEMS WITH NON-SMOOTH SOLUTIONS USING FINITE ELEMENT METHOD

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The problem of numerical modeling of stress-strain state of rock masses in the cases of non-smooth rheological properties of these masses and non-smooth boundary conditions is considered. The problem emerges from the need to estimate the potential drilling risks at the early stages of hydrocarbon field development. Conventionally, at the early stages of HC field development we estimate the drilling risks based on the preliminary mechanical models built from the exploration seismic data. From the interpretation of seismic data, we get either the models of continuous properties (e.g. the results of conventional seismic inversion) or the structural models that describe the configuration of layer boundaries. Estimates of the elastic and mechanical properties may be assigned to the geological layers and objects in the structural models. In that case, the models of mechanical properties of the subsurface have discontinuous boundaries. The current study is focused on such discontinuous models of mechanical properties of rocks. Usage of such models leads to the need to state boundary conditions as discontinuous functions within the framework of geomechanical modeling. Hence, standard numerical modeling techniques should be revisited so that they can incorporate discontinuous (non-smooth) mechanical models with non-smooth boundary conditions. The study presents the results of the geomechanical modeling for discontinuous models of the mechanical properties built from the reflection seismic data acquired in the Russian Arctic shelf. The estimation of stress-strain state of rocks is completed for several models that contain typical geological objects associated with potential risks for the offshore drilling in the research area. Finite element method is applied to compute the stresses in the models that contain permafrost and gas-bearing intervals in the near-surface. Numerical calculations are carried out using Fidesys computational software. It is shown that discontinuous models of mechanical properties require adjustments in the numerical modeling approach. Discontinuous spectral elements are needed to properly simulate stresses and strains fields in such models

Keywords: reservoir geomechanics, drilling risks assessment, mechanical properties, computational mechanics, Finite Element Method (FEM).

Introduction

Development of offshore oil and gas fields is associated with significant technological difficulties. One of the tasks of geophysical exploration is to assess and mitigate potential risks for drilling and offshore construction. Development of the hydrocarbon fields in Arctic and other marginal seas is essential for producing oil resources and ensuring the energy safety of Russia. At the same time, development of such fields is characterized by considerable technological difficulties and increased drilling risks. Assessment of these risks generally suggests analysis of various processes emerging during drilling and their features related to the properties of the geological media in the considered locations. Current study is devoted to the geomechanical aspects of drilling risks assessment.

Reservoir geomechanics is among the major disciplines dealing with drilling risks [*Zoback,* 2007]. Geomechanical modeling allows one to assess potential drilling risks such as wellbore breakouts, emergence and development of tensile fractures in the near wellbore formations. Potential drilling risks are controlled by mechanical properties of rocks, stresses acting in the subsurface and drilling conditions (drilling mud weight). To avoid drilling risks, it is necessary to evaluate the values of drilling mud weight that are safe for drilling. In other words, it is necessary to estimate values of drilling mud weight that will prevent formations from breakouts into the borehole and development of tensile fracture during drilling.

Typical methods for assessment of potential drilling risks rely on the results of geomechanical modeling [*Kwakwa, Batchelor, Clark,* 1991]. These methods generally include three steps. Firstly, the model of mechanical properties of rocks is constructed based on the analysis of seismic data, well logs, and core experiments. Secondly, the stress state of rocks is estimated as the result of either theoretical studies [*Amadei,* 1984] or numerical modeling [*Wang, Dusseault,* 2003; *Aadnoy, Belayneh,* 2004]. Finally, the stress state and the information on the rocks strength properties are used to estimate safe drilling conditions [*Maleki et al.,* 2014]. The discussed steps can be completed under various assumptions. For instance, estimation of the drilling mud weight that is safe for drilling can be completed for elasto-plastic media [*Aadnoy, Belayneh,* 2004] with various constitutive models [*Coelho et al.,* 2005] that can incorporate natural fractures in the rocks [*Yamamoto, Shioya, Uryu,* 2002; *Karatela et al.,* 2015]. Thermal and poroelastic effects also can be considered when computing the effective stresses [*Wang, Dusseault,* 2003; *Liu et al.,* 2016]. It should be mentioned that the dual-porosity approach appears as a useful tool for estimation of the drilling risks in fractured rocks [*Zhang, Bai, Roegiers,* 2006].

Despite the fact that the general workflow of drilling risks assessment based on the results of geomechanical modeling is well formulated [*Kwakwa, Batchelor, Clark,* 1991] there could be problems with its realization in practice. First, results of drilling risks assessment strongly depend on the quality of the model of mechanical properties. The uncertainties in the mechanical model introduce bias into the estimates of the drilling mud weight critical values. Hence, specific methods that accurately take these uncertainties into account during geomechanical modeling should be applied [*Delgado, Kumar,* 2015]. These methods facilitate assessment of risks in terms of probabilities [*Moos et al.,* 2003].

The other problem is related to the quantitative issues arising during numerical modeling of the stress fields. These issues are related to discontinuities of the fields of mechanical properties and boundary conditions used for geomechanical modeling. The need to solve the basic equations of poroelasticity, i.e. equations of equilibrium and constitutive equations, results in numerical discontinuities, complicating the modeling process [*Karatela et al.,* 2015]. Two basic approaches to solving these issues include dealing with the discrete element models [*Yamamoto, Shioya, Uryu,* 2002] and the construction of effective models of mechanical properties of rocks [*Garagash et al.,* 2015]. Both ways have their limitations. The discrete element models focus on the rock mass discontinuities, e.g., natural fractures and faults, while the effective models are constructed after the specific smoothing of the mechanical fields. Discrete element models provide reliable results when data on geological discontinuities are of high quality, while the effective models are not that demanding, but, in turn, only provide information related to averaged properties of rock masses and stress-strain fields. It should be mentioned that most studies being published follow these two approaches. The software packages that are usually used in practice to calculate stress and strain fields in rock masses for the purpose of drilling risks assessment are based on the finite element method and the discrete difference method [*Zoback,* 2007].

Geomechanical modeling for drilling risks assessment in offshore conditions is complicated by both types of the problems. High cost of drilling of the offshore wells makes the exploration drilling utterly rare. Thus, geophysical data become the main source of information on the mechanical properties of rocks being studied. Lack (or absence) of exploration wells and cores extracted for laboratory experiments leads to the poor quality of the constructed models of mechanical rock properties. But these models are essential for the evaluation of the safe drilling conditions. The mechanical models of high quality are required for the prediction of risks in the areas characterized by complex geological structures and presence of layers under anomalously high pressure (typical drilling risks).

The second typical problem is that the spatial distributions of mechanical properties in the models built from exploration seismic data can be non-smooth. In fact, two ways of dealing with seismic data can be highlighted: the results of seismic data interpretation may be either presented in a form of continuous fields of dynamic elastic properties, or distinct regions characterized by constant properties distribution within each region. The second way of dealing with seismic data is based on geological reasons, geometrical regions with constant properties are associated with distinct layers formed under different geological conditions. The current study is focused on the problem of dealing with results of seismic data interpreted in this way. The discontinuous fields of mechanical properties lead to the need to set boundary conditions (generally on stresses acting at the imaginary bounds of the model) as discontinuous functions of coordinates to satisfy the Saint-Venant's compatibility conditions. Consequently, the boundary conditions and the material properties of the rocks are non-smooth that results in problems for the numerical modeling of the stress state. Even though the existing numerical methods (finite difference, finite element, and discrete element methods) provide solutions that allow overcoming these difficulties, the obtained geomechanical models remain imprecise in the zones of non-smooth mechanical properties of the fields [*Wang, Dusseault,* 2003; *Liu et al.,* 2016].

The current study presents an approach that allows one to overcome the discussed above problem. The approach is a discontinuous spectral element method of geomechanical modeling. Contrary to the discrete element method, the finite element method, and the finite difference method, which are widely used in practice, the discontinuous spectral element method is not a conventional tool to deal with the reservoir geomechanics problem. At the same time, it is capable of handling non-conformal high-order discretizations that facilitates correct simulation of the discontinuous fields [*Kukushkin et al.,* 2019]. The study reveals the advantages of the discontinuous spectral element method for the drilling risks assessment on example of the offshore hydrocarbon field at Russian continental shelf.

The borehole drilling in the research area can be complicated due to the presence of permafrost zones and zones of increased gas saturation in the near-surface. The quantitative interpretation of the seismic data allowed us to construct the models of the mechanical properties of rocks on the area. The numerical geomechanical modeling using the discontinuous spectral element method provided estimates of the stresses acting in the zones of potential drilling risks. The numerical calculations were carried out using CAE Fidesys computational software [*Morozov, Levin, Vershinin,* 2015; *Karpenko et al.,* 2016]. The stress and strain fields obtained as the result of the geomechanical modeling were used for the drilling risks assessment in the research area.

Model of mechanical properties

The continental Russian shelf in the Arctic is characterized by complex geological conditions and presence of the near-surface geohazards that represent potential risks for the drilling. The typical geohazards are saturated gas deposits, zones of seasonal freezing and permafrost areas in the upper part of the geological section (first hundreds of meters from the seabed). These potentially hazardous geological objects are ubiquitously identified on the inner shelf of the Laptev Sea, the Pechora Sea and other water areas of the central and eastern Arctic according to the seismic and engineering drilling data [*Bogoyavlensky,* 2012].

In the paper, we consider a research area in the shelf of the Laptev Sea. In this area, the models of the near-surface have been developed based on the interpretation of the reflection seismic data acquired on a grid of 2D regional profiles. The seismic observations were carried out in several frequency ranges, which facilitated a propagation depth of several hundred meters from the seabed with a vertical resolution of 0.5–7 m. The research area is characterized by water depths of 30–70 m [*Kolyubakin et al.,* 2016].

Analysis of the seismic wave patterns revealed the presence of numerous amplitude anomalies in the upper part of the geological section. These anomalies are mainly associated with the presence of gas-saturated sediments and frozen soils. The differentiation between these two types of anomalies is complicated due to their similar manifestation in the seismic wavefield. The gas-saturated deposits and the objects of a cryogenic genesis are characterized by the intense reflection amplitudes, uneven geometry of the reflecting boundaries, numerous diffractors, and inhomogeneous distribution in the subsurface. The situation is complicated since a large amount of natural gas can be released due to the seasonal thawing of the frozen deposits [*Lobkovskiy et al.,* 2015]. Figure 1 presents the fragments of a seismic section with the interpretation of the possible permafrost zone (*on the left*) and the possible gas inclusion (*on the right*).

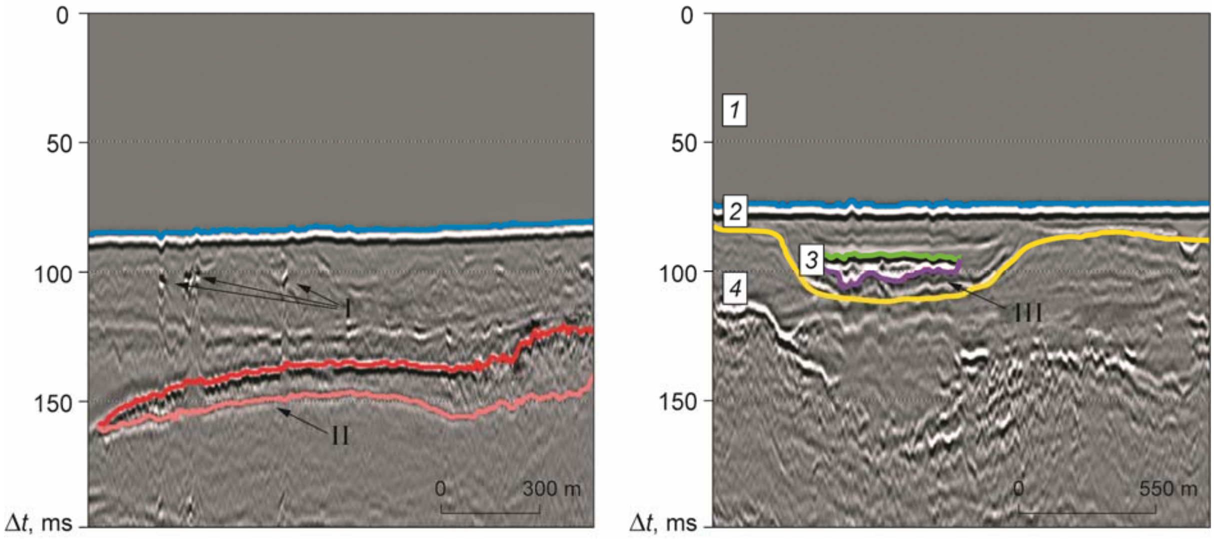

Fig. 1. Fragments of seismic section with interpretation of possible permafrost zone (*on the left*) and possible gas inclusion in the paleochannel (*on the right*). On horizontal axes – two-way travel time Δ*t*, ms; *1–4* – layers used in numerical simulation; I – gas outlets; II – permafrost lens; III – gas-saturated inclusions. Lines of different colors are marked: the seabed (blue); upper (red) and lower (pink) boundaries of the permafrost lens; lower boundary of the paleochannel (yellow); upper (green) and lower (purple) boundaries of the gas cap

Рис. 1. Фрагменты сейсмического разреза с интерпретацией возможной зоны вечной мерзлоты (*слева*) и возможного включения газа в палеоканале (*справа*). На горизонтальных осях – двойное время пробега Δ*t*, мс; *1–4* – слои, используемые при численном моделировании; I – выходы газа; II – линза вечной мерзлоты; III – газонасыщенные включения. Линиями разного цвета обозначены: морское дно (синяя); верхняя (красная) и нижняя (розовая) границы линзы вечной мерзлоты; нижняя граница палеоканала (желтая); верхняя (зеленая) и нижняя (фиолетовая) границы газовой шапки

Based on the interpretation of the seismic data two elastic models with the inclusions in the paleochannel have been developed. In the first model, the inclusion corresponds to the gas-saturated soils characterized by the low *P*-wave velocities and the decreased V_P/V_S ratio relative to the embedding deposits. In the second model, the inclusion with the same configuration of reflecting boundaries is characterized by high *P*-wave velocity and density, which is typical for the frozen deposits. The elastic properties for the two models were obtained from the *P*-wave velocity analysis and the seismic inversion [*Pirogova et al.,* 2019].

Various correlations may be used to transform the spatial distributions of the elastic wave velocities to the distributions of static and dynamic elastic moduli. Typically, these correlations are established based on the results of laboratory experiments on rock samples.

In the current study, the laboratory studies on rock samples were unavailable. Due to the lack of the experimental data it has been suggested to construct imitational mechanical models of the subsurface. The seismic datasets are considered as a basis for construction of the imitational models of the static elastic moduli. The geometry of the objects in the subsurface and dynamic elastic moduli are estimated from the seismic data. A set of empirical parameters that establishes the relations between the dynamic elastic moduli and the static elastic moduli corresponds to the unique imitational model of the mechanical properties. Analysis of the relationships between these empirical parameters and the resultant drilling risks can be completed using a system analysis approach [*Dubinya et al.,* 2020] and remains among the perspectives of the current study.

The simplest models of elastic properties are considered in the current study: Figs. 2–5 represent spatial distributions of the static elastic moduli for these models.

The fields of mechanical properties are clearly non-smooth. In fact, each zone shown in its own color represents its own mechanical facie characterized by constant mechanical properties. Any facie is considered as a homogeneous isotropic medium with its elastic properties deduced to the two independent elastic moduli.

Although this model may seem oversimplified, it is, in fact, the worst scenario for the numerical modeling of the stress fields as these fields are non-differentiable near the interfaces between the facies.

Dynamic elastic Lame parameters λ and μ are determined using the following well-known relations based on the density ρ and velocities of longitudinal and shear waves V_P and V_S respectively [*Sedov,* 1970]

$$\mu = \rho V_S^2; \quad \lambda = \rho V_P^2 - 2\mu . \qquad (1)$$

The distributions of Lame parameters obtained from these equations for the two considered geomechanical models are shown in the figures below (see Figs. 2–5). These data are consequently used to reconstruct corresponding spatial distributions of static elastic moduli following the empirical relationships typical for the characterized rocks.

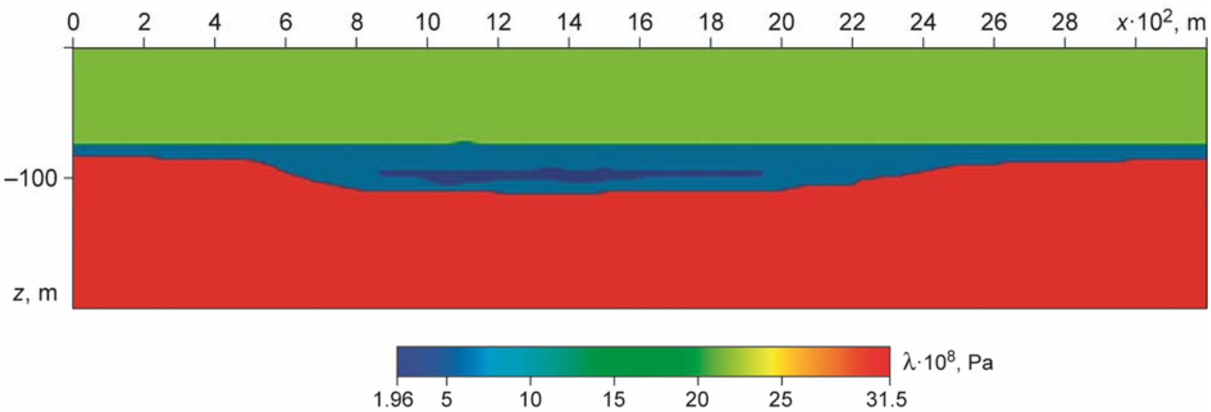

Fig. 2. The distribution of the first Lame parameter (λ, Pa) for the model with gas inclusion

Рис. 2. Распределение первого параметра Ламе (λ, Па) для модели с включением газа

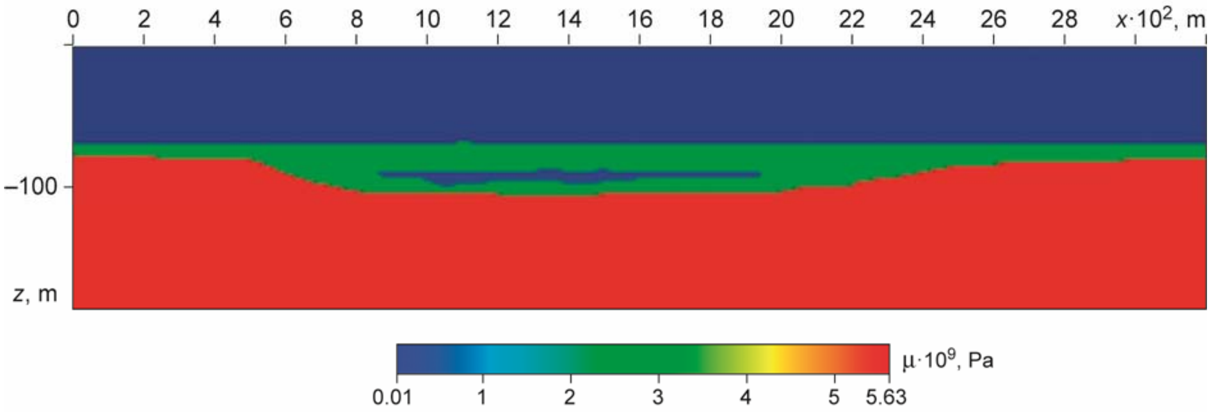

Fig. 3. The distribution of the shear modulus (μ, Pa) for the model with the gas inclusion

Рис. 3. Распределение модуля сдвига (μ, Па) для модели с включением газа

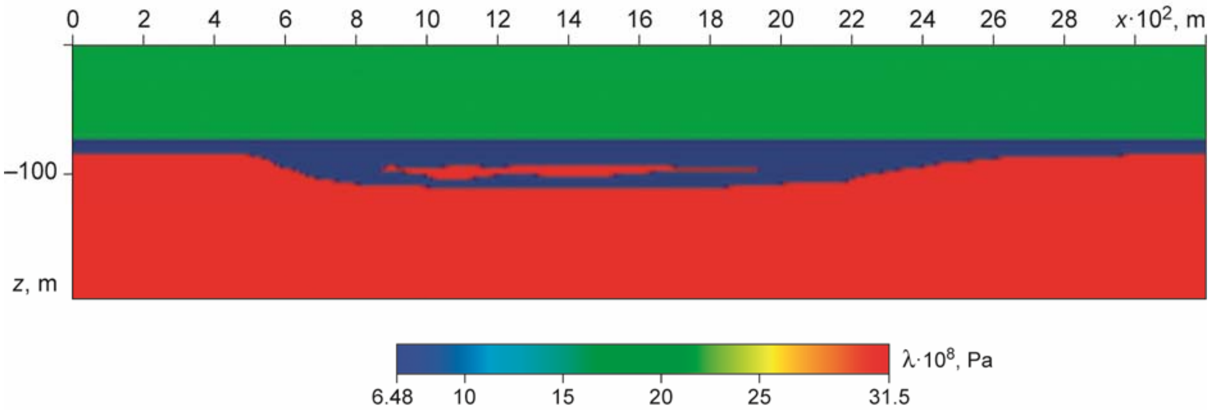

Fig. 4. The distribution of the first Lame parameter (λ, Pa) for the model with the permafrost inclusion

Рис. 4. Распределение первого параметра Ламе (λ, Па) для модели с включением вечной мерзлоты

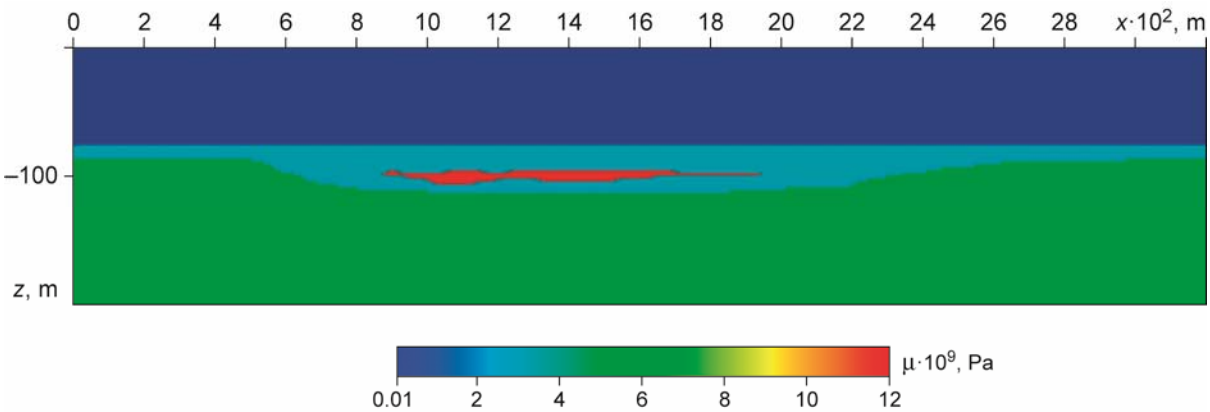

Fig. 5. The distribution of the shear modulus (μ, Pa) for the model with permafrost inclusion

Рис. 5. Распределение модуля сдвига (μ, Па) для модели с включением вечной мерзлоты

These models of mechanical properties can be directly used for numerical modeling of the stress and strain fields in the area after the proper introduction of boundary conditions.

Geomechanical modeling

With the models of mechanical properties being established, geomechanical modeling can be carried out after setting the boundary conditions. It is necessary to define horizontally directed stresses acting far away from the object as functions of depth to properly set the boundary conditions related to tectonic forces.

According to the empirical studies of stress state in the upper layers of Earth's crust [*Hoek, Brown,* 1980; *Sheorey,* 1994], the average horizontal stress – the half-sum of horizontal stresses $(\sigma_{xx} + \sigma_{yy})/2$ – acting in the upper layers of the Earth's crust remains within the certain limits with dependence on depth z and vertical stress σ_{zz}. The relationship between the average horizontal stress and the vertical stress has to be inside the interval:

$$\frac{\sigma_{xx} + \sigma_{yy}}{2\sigma_{zz}} \in \left[\frac{100}{z} + 0.3; \quad \frac{1500}{z} + 0.5\right]. \tag{2}$$

While these equations are typically used for constraining horizontal stresses for lower depths (up to several kilometers), they remain valid for the depths typical for the objects considered in the current study.

On the other hand, there is a condition of stable seismic situation in the region, which can be expressed with the usage of linear Mohr-Coulomb criterion with zero cohesion on the optimally oriented faults [*Zoback,* 2007]:

$$\frac{\sigma_1}{\sigma_3} \le \frac{1 + \sin\varphi}{1 - \sin\varphi}, \tag{3}$$

where σ_1 and σ_3 are the maximum and minimum principal stresses acting in the rock mass respectively; φ is its internal friction angle.

Equations (2) and (3) can be combined to set horizontal stresses as inverse functions of depth. The magnitudes of stresses determined from these functions should, on one hand, stay within the limits (2), and, on the other hand, never exceed the condition (3) in any element of the geomechanical model. As a result, the following function is suggested as a boundary condition set on horizontal stresses: a horizontally oriented force distributed along the lateral surfaces of the model as:

$$N(z) = \begin{cases} 0, & z < z_{seafloor}, \\ \rho_b g z\left(\dfrac{b}{z} + c - \dfrac{\nu}{1-\nu}\right), & z \ge z_{seafloor}. \end{cases} \tag{4}$$

Here $N(z)$ is horizontally directed distributed force acting on the lateral surface of the model, ρ_b is rock density, $z_{seafloor}$ is seafloor depth, ν is Poisson's ratio. b and c are parameters typical for the current object – variation of these parameters provides an opportunity to analyze drilling risks at areas with different tectonic states. The element $\nu/(1 - \nu)$ is related to the soil back pressure (which would have been taken into account if geomechanical modeling had been carried out with zero horizontal displacements set on lateral sides of the model).

Note that $N(z)$ is non-smooth function by definition. Moreover, parameters b and c, as well as density and Poisson's ratio, are step functions: they are constant only within a certain facie.

The equations listed above would be valid for a medium with no fluid saturation, yet it is evident that real rocks are characterized by presence of fluid with certain pressure referred to as pore pressure below. It is a general practice to use Biot poroelasticity model to deal with fluid-saturated rocks [*Biot,* 1956]. According to this model, any volume of rock mass is characterized by effective stress, equal to the difference between total stress and pore pressure which is multiplied by a certain coefficient known as Biot coefficient α. This coefficient stays between zero and one for real rocks, so it is necessary to get its value in order to perform any sort of geomechanical modeling. There are various ways to predict Biot coefficient from field and experimental data [*Franquet, Abass,* 1999] and theoretical modeling [*Cho et al.,* 2016]. It is essential to know elastic properties of rock itself (Equations (1)) and elastic properties of the solid phase and compressibility of fluid. Estimation of these parameters is not within the scope of the current study – here Biot's coefficient is assumed to be taken equal to one. Nevertheless, whenever any method is applicable to estimate Biot's coefficient, parameters of this method may be considered as parameters of the whole model in the same way as coefficients *b* and *c* in Equations (4).

It is important to mention that pore pressure may be taken into account in two different ways with respect to the loading type – be it drained or undrained loading. Undrained case is considered in the current study.

Finally, equations (4) should be altered to take pore pressure *P* into account. While distributed force $N(z)$ acts on unsaturated medium, the corresponding case of saturated medium with fluid pressure *P* suggests higher force $N_f(z)$:

$$N_f(z) = \begin{cases} 0, \quad z < z_{seafloor}, \\ \rho_b g z\left(\dfrac{b}{z} + c - \dfrac{\nu}{1-\nu}\right) + P\dfrac{1-2\nu}{1-\nu}, \quad z \ge z_{seafloor}, \end{cases} \tag{5}$$

for the considered case.

Before going into the details of numerical modeling scheme let us present the mathematical model, which was used to describe distributions of stresses and strains in heterogeneous media with properties obtained from the results of seismic interpretation as it was described above. Poroelastic model according to Biot–Terzaghi law [*Biot,* 1956]:

$$\nabla \cdot \left(\vec{\vec{\sigma}}\left(\vec{u}\left(\vec{x}\right)\right) - \alpha p\left(\vec{x}\right)\vec{\vec{I}}\right) + \vec{f}\left(\vec{x}, t\right) = 0 . \tag{6}$$

Equation (6) represents equilibrium equation in the domain **Ω**. Here $\vec{\vec{\sigma}}$ is stress tensor for the poroelastic solid; $\vec{u}\left(\vec{x}\right)$ is displacement vector field; α is poroelastic Biot–Terzaghi coefficient; $p\left(\vec{x}\right)$ is external pore pressure; $\vec{\vec{I}}$ is identity tensor; $\vec{f}\left(\vec{x}, t\right)$ represents external volumetric forces (i.e. gravity) depending on both spatial $\vec{x}$ and temporal *t* coordinates.

Boundary conditions may be considered either for displacements at the external domain's boundary Γ_u (7) or for traction at the external domain's boundary Γ_p (8):

$$\vec{u}\left(\vec{x}\right)\Big|_{x\in\Gamma_u} = \vec{u}_{\Gamma_u} , \tag{7}$$

$$\vec{\vec{\sigma}}\left(\vec{x}\right)\cdot\vec{n}\Big|_{x\in\Gamma_p} = \vec{\vec{\sigma}}_{\Gamma_p} \cdot \vec{n} . \tag{8}$$

It is necessary to consider constitutive laws to solve equation (6). Hooke's law (9) is used for this purpose in the current study:

$$\vec{\vec{\sigma}} = \lambda\left(\vec{\vec{\varepsilon}} : \vec{\vec{I}}\right)\vec{\vec{I}} + 2\mu\vec{\vec{\varepsilon}}, \tag{9}$$

where strain tensor $\overleftrightarrow{\varepsilon}$ is introduced following small deformation theory:

$$\overleftrightarrow{\varepsilon} = \frac{1}{2}\left(\nabla\otimes\vec{u} + (\nabla\otimes\vec{u})^T\right). \tag{10}$$

The given analytical partial differential equations are discretized numerically using finite element method (FEM) [*Zienkiewicz, Taylor,* 2014].

The domain **Ω** (representing a rectangle given by its space coordinates) is discretized using structured mesh of $N_x \times N_y$ linear quadrangular elements (QUAD4) so that the FEM mesh is coincident with the initial seismic interpretation grid. Correspondingly, each quadrangular finite element possesses its own elastic properties inherited from the seismic inversion in the coinciding cell with the same coordinates. The boundary Γ is discretized accordingly by the edges of the quadrangles. An example of such a mesh is given in Fig. 6 (different colors correspond to different elastic properties).

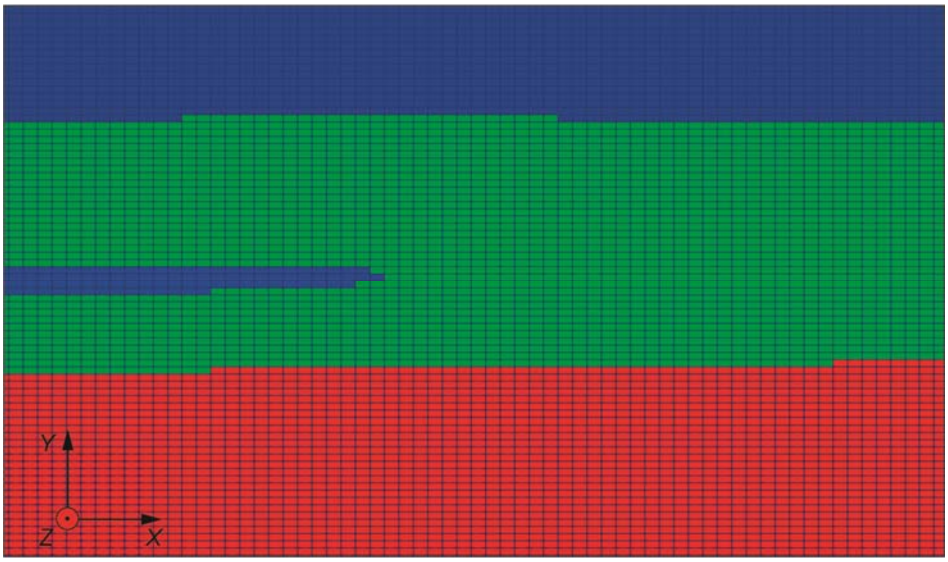

Fig. 6. Finite element mesh for numerical modeling

Рис. 6. Конечно-элементная сетка для численного моделирования. Разным цветом отмечены области с разными механическими свойствами

Application of Galerkin method [*Galerkin,* 1915; *Zienkiewicz, Taylor,* 2014] to the equilibrium equation (6) gives:

$$\int_\Omega N_i \nabla \cdot \left(\overleftrightarrow{\sigma} - \alpha p \vec{I} \right) d\Omega + \int_\Omega N_i \vec{f} d\Omega = 0 \,, \tag{11}$$

where N_i are form functions. We rewrite this taking into account the boundary conditions and using Green's formula:

$$\int_\Gamma N_i \overleftrightarrow{\sigma_\Gamma} \cdot \vec{n} d\Omega - \int_\Omega \nabla N_i \cdot \left(\overleftrightarrow{\sigma} - \alpha p \vec{I} \right) d\Omega + \int_\Omega N_i \vec{f} d\Omega = 0 \,. \tag{12}$$

We separate the pore pressure term and replace gravity with volume force:

$$\int_\Omega \nabla N_i \cdot \vec{\sigma} d\Omega - \int_\Gamma N_i \overleftrightarrow{\sigma_\Gamma} \cdot \vec{n} d\Omega - \int_\Omega \nabla N_i \cdot \alpha p \vec{I} d\Omega - \int_\Omega N_i \rho \vec{g} d\Omega = 0 \,. \tag{13}$$

We assume that the displacements are approximated using the QUAD4 shape functions in each element by:

$$u \approx \hat{u} = \sum_a N_a \left(\vec{x} \right) \vec{u}_a^e \,. \tag{14}$$

Isoparametric elements [*Zienkiewicz, Taylor,* 2014; *Levin et al.,* 2013] for the posed problem's discretization in space were used to approximate unknown fields inside the finite element mesh and compute the listed integrals numerically.

We represent the strain tensors approximation in a matrix form:

$$\boldsymbol{e} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \sum_a \boldsymbol{B}_a \cdot \vec{u}_a \,, \tag{15}$$

where

$$\boldsymbol{B}_a = \begin{bmatrix} \dfrac{\partial N_a}{\partial x} & 0 \\ 0 & \dfrac{\partial N_a}{\partial y} \\ \dfrac{\partial N_a}{\partial y} & \dfrac{\partial N_a}{\partial x} \end{bmatrix}. \tag{16}$$

Stress tensor and Hooke's law (9) may be now written in matrix form as well:

$$\boldsymbol{\sigma} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \boldsymbol{D} \cdot \boldsymbol{\varepsilon}, \tag{17}$$

where $\boldsymbol{D}$ is a matrix of elasticity coefficients:

$$\boldsymbol{D} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & 1-\nu & 0 \\ 1-\nu & 1-\nu & 0 \\ 0 & 0 & (1-2\nu)/2 \end{bmatrix}. \tag{18}$$

After substituting this approximation in (13) we get:

$$\sum_i \left[\int_{\Omega_i} \boldsymbol{B}^T{}_i \boldsymbol{D} \sum_j \boldsymbol{B}_j \cdot \vec{u}_j d\Omega - \int_{\Omega_i} \boldsymbol{B}^T{}_i \left(\alpha p \vec{I} \right) d\Omega - \int_{\Omega_i} N_i \rho \vec{g} d\Omega - \int_{\Gamma_{\vec{\sigma}_i}} N_i \vec{\sigma}_\Gamma \cdot \vec{n} d\Gamma \right] = 0. \tag{19}$$

We can introduce stiffness $\boldsymbol{K}^e$ for integrals over finite elements as:

$$\boldsymbol{K}^e = \int_{\Omega_e} \boldsymbol{B}^T \cdot \boldsymbol{D} \cdot \boldsymbol{B} d\Omega, \tag{20}$$

where

$$\boldsymbol{B} = \left[\boldsymbol{B}_1, \boldsymbol{B}_2, \dots \boldsymbol{B}_8 \right]. \tag{21}$$

External forces acting at elements are combined of forces at finite elements due to gravity $\vec{F}^e_{g_a}$:

$$\vec{F}^e_{g_a} = \int_{\Omega^e{}_a} N_a \rho \vec{g} d\Omega^e{}_a, \tag{22}$$

external forces at finite elements due to pore pressure $\vec{F}^e_{p_a}$:

$$\vec{F}^e_{p_a} = \int_{\Omega^e{}_a} \boldsymbol{B}^T{}_a \cdot \alpha p \vec{I} d\Omega^e{}_a, \tag{23}$$

and external forces at boundaries of finite elements due to prescribed normal stresses at the domain's boundary $\vec{F}^e_{\sigma_a}$:

$$\vec{F}^e_{\sigma_a} = \int_{\Gamma^e{}_a} N_a \vec{\sigma}_\Gamma \cdot \vec{n} d\Gamma^e{}_a. \tag{24}$$

To take prescribed boundary displacements into account, one needs to modify global stiffness matrix $\boldsymbol{K}$. Firstly, we calculate nodal force $\vec{F}_u$ from the given nodal displacements $\vec{u}_\Gamma$ as:

$$\vec{F}_u = \boldsymbol{K} \cdot \vec{u}_\Gamma. \tag{25}$$

Next we zero a row and a column of the stiffness matrix corresponding to the fixed degree of freedom, wherein diagonal element stays unmodified. The system with a modified matrix $\boldsymbol{K}'$ takes form:

$$\boldsymbol{K}' \cdot \vec{u}_\Gamma = \vec{F} - \vec{F}_u, \tag{26}$$

so that the resulting sparse system of linear algebraic equations is:

$$\boldsymbol{K} \cdot \vec{u} = \vec{F}, \quad (27)$$

where $\boldsymbol{K} = \sum_e \boldsymbol{K}^e$ is a global stiffness matrix obtained by assembling local stiffness matrices for each finite element Ω^e; and $\vec{F} = \sum_e \vec{F}_\sigma^e + \sum_e \vec{F}_p^e + \sum_e \vec{F}_g^e$ is a global force vector obtained by assembling local force vectors for each finite element Ω^e and boundary element Γ^e.

The obtained sparse system of linear algebraic equations is solved using a direct sparse solver [*Morozov, Levin, Vershinin,* 2015; *Zienkiewicz, Taylor,* 2014]. The resultant displacement field is used further to compute strain and stress fields.

Results

Geomechanical simulations were performed in CAE Fidesys software [*Morozov, Levin, Vershinin,* 2015] based on the discontinuous finite and spectral element method [*Morozov, Levin, Vershinin,* 2015; *Karpenko et al.,* 2016; *Kukushkin et al.,* 2019]. The figures below show the distributions of the principal stresses and the von Mises stress intensity in the geomechanical models with the inclusions of gas (Figs. 7, 8) and permafrost (Figs. 9, 10).

We note regions of high stress concentration observed at the boundaries of inclusions, as well as at the boundaries of geological layers, caused by a sharp changes in the elastic moduli. The finite element method, which is based on the variational (so-called weak) formulation of the boundary value problem of the theory of poroelasticity [*Zienkiewicz, Taylor,* 2014], made it possible to correctly model these discontinuities in the material parameters, setting continuous modules inside each element, while discontinuities pass along the boundaries between the elements [*Levin et al.,* 2013]. Thus, the obtained numerical solution admits discontinuities in stresses between the finite elements, while maintaining continuity of the main unknowns of the boundary problem (displacements) at the boundaries between the elements [*Kukushkin et al.,* 2019], which fully corresponds to the geomechanical nature of the considered problem.

It can be seen from the Figs. 7–10 that despite the initial fields of mechanical properties being discontinuous, the resultant stress fields are continuous (i.e., normal stresses are continuous along the normal to the interface between the layers). In fact, they satisfy the differential equations of mechanical equilibrium. The mentioned techniques of smoothing the mesh near the boundaries between the domains with different mechanical properties made it possible to get this solution from the numerical modeling using the finite element method.

Comparison of the figures related to two types of potential zones of increased drilling risks reveals the difference between their effects on stress distributions. In fact, although gas and permafrost inclusions may seem similar at seismic data, stresses redistributions near their boundaries are different. Moreover, these differences are strongly nonlinear: gas inclusions are characterized by higher magnitudes of the first principal stress within the inclusion, but with lower stress intensity, compared to the permafrost inclusions. This difference has its clear effect on drilling because the critical values of mud weight are strongly dependent on stresses. According to the obtained results, one may expect the same widths of drilling mud density window for these two inclusions. However, the mud density that leads to the breakouts of formations into the wellbore is lower for the permafrost inclusion, which results from the qualitative analysis of the main tendencies in mud weight dependencies on stresses [*Delgado, Kumar,* 2015].

Nevertheless, it should be noted that relationships between elastic moduli and strength properties – another major factor influencing safe drilling mud window – can be different for the considered types of inclusions [*Etesami, Shahbazi,* 2014], so the aforementioned tendency

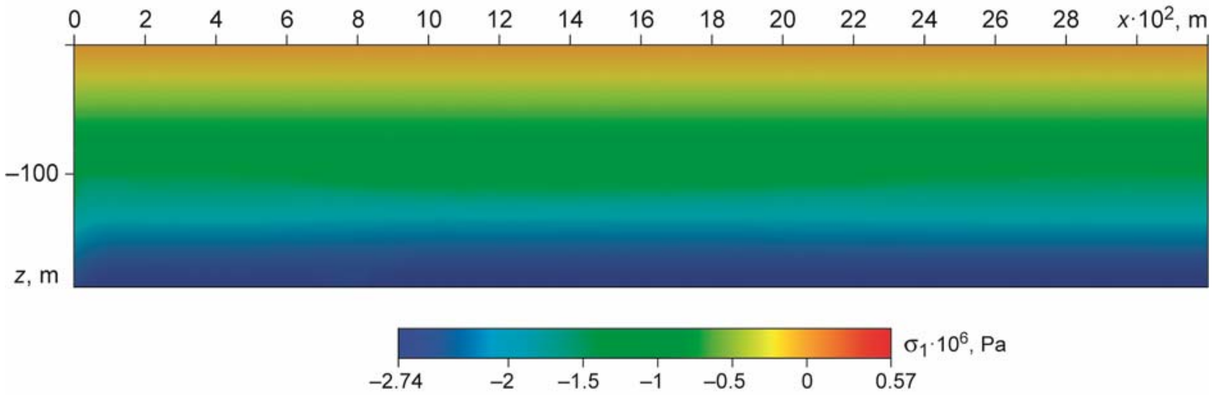

Fig. 7. The distribution of the first principal stress (σ_1, Pa) for the model with the gas inclusion. Compressive stresses are negative

Рис. 7. Распределение первого главного напряжения (σ_1, Па) для модели с включением газа. Сжимающие напряжения приняты отрицательными

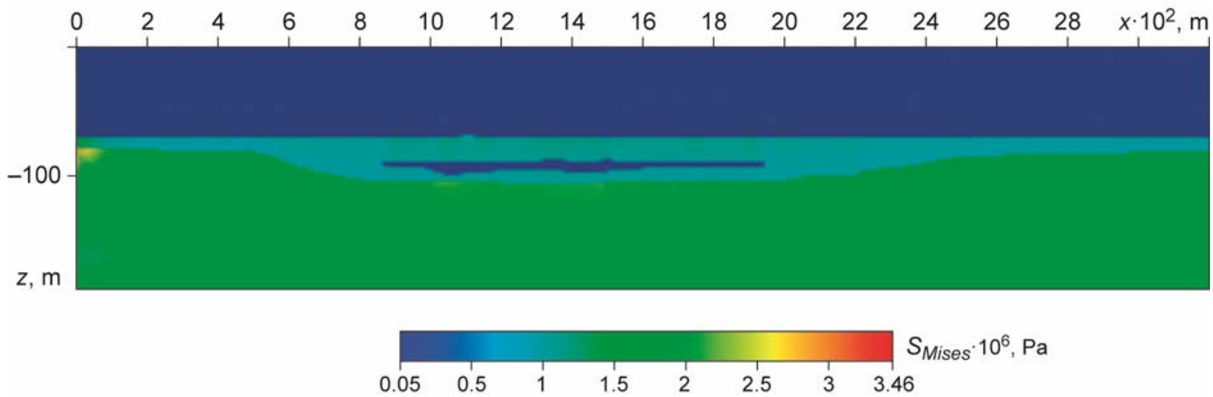

Fig. 8. Von Mises stress intensity distribution (S_{Mises}, Pa) for the model with the gas inclusion

Рис. 8. Распределение интенсивности напряжений по Мизесу (S_{Mises}, Па) для модели с включением газа

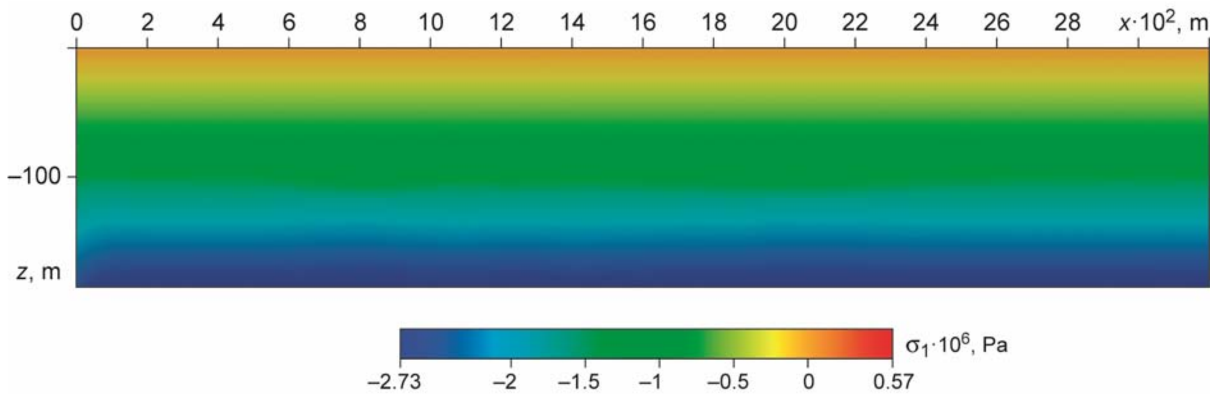

Fig. 9. Distribution of the first principal (compressive) stress (σ_1, Pa) for the model with permafrost inclusion. Compressive stresses are negative

Рис. 9. Распределение первого главного напряжения (σ_1, Па) для модели с включением вечной мерзлоты. Сжимающие напряжения приняты отрицательными

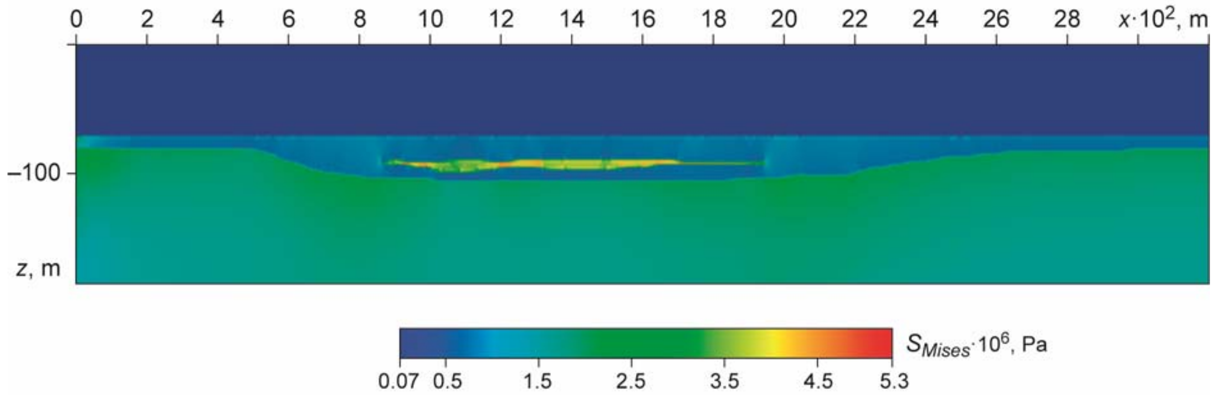

Fig. 10. Von Mises stress intensity distribution (S_{Mises}, Pa) for the model with permafrost inclusion

Рис. 10. Распределение интенсивности напряжений по Мизесу (S_{Mises}, Па) для модели с включением вечной мерзлоты

is not a strict rule. But this rule should be kept in mind during initial assessment of drilling risks at the earlier stages of offshore fields development before any data from well logs and laboratory experiments are obtained.

Conclusions

The presented results highlight the main steps of numerical geomechanical modeling for drilling risks assessment at the earlier stages of offshore fields development. These steps include analysis of seismic data (see Fig. 1), construction of dynamic and static elastic moduli models (see Fig. 2–5), setting the boundary conditions at equation (5), and estimation of stress fields in the studied area (see Fig. 7–10).

The presented workflow is used for drilling risks assessment and calculation of critical densities of mud weight. Usage of the drilling mud of the density that falls in the estimated safe drilling mud density window results into the safe drilling.

There are two principal problems that complicate the offshore drilling risks assessment. Firstly, the essential data on the mechanical properties of rocks are very limited in the offshore areas. High costs of offshore drilling exploration make well-established approaches to the drilling risks assessment utterly impractical. At the same time, high quality seismic data, which are typically available at the offshore fields, provide an opportunity to construct detailed models of dynamic elastic moduli of the subsurface. In turn, these models can be utilized for the building of imitational models of the static elastic moduli and strength properties of the rocks.

Despite the lack of other essential data (e.g., laboratory studies of rock samples) that are required for estimation of precise values of the safe drilling mud weight, general geomechanical tendencies can still be observed from the imitational models. The obtained results can be used as the initial idea on the drilling conditions in the research area. Note that these results come from the preliminary drilling risks assessment. Introduction of additional data such as well logs and laboratory experiments will clarify the recommendations on the drilling mud density.

The other problem addressed in the paper is related to non-smooth fields of mechanical properties and boundary conditions. This problem is mainly related to numerical modeling as non-smooth functions demand handling of non-conformal high-order discretizations to correctly simulate discontinuous fields of stresses and strains. These discontinuities result into imprecise solutions near the interfaces between mechanical facie if conventional methods of numerical modeling – the finite element, the discrete element, the finite differences methods – are used to simulate stress-strain states. The current study reveals the advantages of the discontinuous spectral element method in overcoming this problem. The resultant stress fields estimated by this method for the discontinuous models of mechanical properties are continuous, satisfying the differential equations of the mechanical equilibrium. Hence, drilling risks assessment with the usage of the discontinuous spectral element method might be more reliable compared to other methods of numerical modeling.

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Conflict of interest

The authors declare they have no conflict of interest

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ОСОБЕННОСТИ ЧИСЛЕННОГО РЕШЕНИЯ ЗАДАЧ ГЕОМЕХАНИКИ МЕСТОРОЖДЕНИЙ С НЕГЛАДКИМИ РЕШЕНИЯМИ МЕТОДОМ КОНЕЧНЫХ ЭЛЕМЕНТОВ

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Аннотация. Рассматривается задача численного моделирования напряжённо-деформированного состояния массивов горных пород для случаев негладких граничных условий и реологических свойств этих пород. Решение таких задач необходимо при оценке потенциальных рисков при бурении на ранних стадиях разработки месторождений углеводородов. Обычно упомянутая оценка рисков выполняется с использованием предварительных механических моделей, построенных по результатам интерпретации данных сейсморазведки. Результаты интерпретации сейсмических данных могут быть представлены либо в виде моделей непрерывных свойств (при традиционной сейсмической инверсии), либо в виде структурных моделей, определяющих конфигурации границ слоёв. Оценки упругих и прочих механических свойств могут быть проведены для каждого геологического слоя и объектов структурной модели. При таком подходе модельные распределения механических свойств массивов горных пород могут претерпевать разрывы. Данное исследование нацелено на работу с такими моделями механических свойств, поля которых имеют разрывы. Применение этих моделей при геомеханическом моделировании ведёт к необходимости постановки граничных условий через негладкие функции. В связи с этим, стандартные методы численного моделирования должны быть пересмотрены таким образом, чтобы иметь возможность воспользоваться разрывными (негладкими) моделями механических свойств и негладкими граничными условиями. В статье представлены результаты геомеханического моделирования, полученные для разрывных моделей механических свойств, которые построены на данных сейсморазведки, проведённой на Российском Арктическом шельфе. Проанализировано напряжённо-деформированное состояние горных пород для нескольких моделей, содержащих в себе геологические объекты, традиционно связываемые с повышенными рисками при бурении на шельфе. Метод конечных элементов использован для расчёта напряжений в моделях, содержащих зоны вечной мерзлоты и включения газа. Численное моделирование выполнено с помощью программного пакета *Fidesys*. Показано, что наличие разрывов в распределениях механических свойств ведёт к необходимости модификации применяемых при моделировании численных методов – для расчёта полей напряжений и деформаций в таких моделях необходимо использование разрывного метода спектральных элементов.

Ключевые слова: геомеханика месторождений, оценка рисков при бурении, механические свойства, вычислительная механика, метод конечных элементов.

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