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GEOPHYSICS

Viscosity of the Earth's Core Based on Seismic Data

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In previous works [12, 13] based on results of the interpretation of the so-called precursors (i.e., seismic waves traveling in the lower part of the Earth's outer core and recorded at the Earth's surface as first arrivals at epicentral distances of 134°–146°), the author of the present paper reported the distribution of longitudinal waves that are characterized by a significant increase in the velocity gradient in the outer core. The new velocity curve obtained compels us to refine the values of the elastic parameters and the viscosity coefficient of the core. Based on analysis of the previous results, the present paper reports bilateral estimates of the elasticity moduli and viscosity coefficient for the outer and inner cores at an oscillation frequency of 1 Hz. The minimum and maximum estimates of the coefficient of dynamic viscosity η vary with depth. In the outer core, the minimum values can vary from $2.0 \cdot 10^8$ to $3.8 \cdot 10^{10}$ Pa s; maximum values, from $2.5 \cdot 10^9$ to $7.5 \cdot 10^{11}$ Pa s. In the F zone, the η value can vary from virtually zero to $4.0 \cdot 10^{10}$ Pa s. In the inner core, the viscosity can vary from $5.0 \cdot 10^{10}$ to 10^{11} Pa s.

It is believed that the Earth's outer core is in a liquid state, because elastic transverse waves cannot pass through this zone. In accordance with this model, the shear modulus in the Earth's outer core is equal to zero. At least, this is the version accepted in modern physical models of the Earth. At the same time, any real liquid possesses viscosity and, consequently, an effective (nonzero) shear modulus relative to the sufficiently high-frequency oscillations.

The viscosity of the Earth's core, particularly the outer core, is least studied. The available theoretical, experimental, and geophysical data on the viscosity of the Earth's outer core show significant discrepancies. According to the majority of theoretical and experimental investigations, melts of iron compounds have a low viscosity (approximately 10^{-2} Pa s) at *PT* conditions corresponding to the outer core [1–4]. At the same time, virtually all geophysical data indicate much higher viscosities (form 10^3 to 10^{12} Pa s) in the Earth's core. For example, according to Zharkov [4], dynamic viscosity in the outer core is sufficiently high, but it does not exceed 10^9 Pa s. According to the data on free core nutation [6], the kinematic viscosity of the outer core is approximately 10^5 St, which corresponds to the value of dynamic viscosity for the outer core (~10^{10} Pa s). Data on the splitting of moda of the Earth's natural oscillations indicate that the viscosity at the outer/inner core interface is equal to $1.22 \cdot 10^{11}$ Pa s [7]. According to the theory of magnetic dynamo in the Earth's core, the viscosity is $2 \cdot 10^7$ St or $2 \cdot 10^{12}$ Pa s [8]. Recently, we obtained data on superrotation of the outer core: owing to tidal forces acting mainly on the Earth's outer shells, the internal core rotates more rapidly than the mantle. Based on the discrepancy between rotation velocities of the internal core and mantle, we can estimate the viscosity of the outer core. Thus, the viscosity of the outer core is estimated at ~10^3 Pa s in [9]. In [10], the kinematic viscosity of the outer core is estimated at 3 M^2/s or ~$3 \cdot 10^4$ Pa s. According to data in [11] based on investigation of the viscosity of liquid metals under pressure up to 10^{10} Pa, the viscosity of melts shows a significant increase along the melt path. Investigations in [11] were based on measurements of the dimensions of crystal grains in samples obtained after the quenching of melts. Extrapolation of results obtained for iron melts to pressures and temperatures existing in the Earth's core suggested that the outer core is composed of melts with viscosity varying from 10^2 to 10^{11} Pa s depending on depth. According to the authors of [11], the outer core is located in an ultraviscous state ($\eta > 10^{11}$ Pa) close to that of glass in physical properties.

Based on previous results pertaining to the distribution of the velocity of longitudinal seismic waves and elastic parameters in the outer core [12, 13], the present paper demonstrates the distribution of the shear modulus and viscosity in the outer and inner cores.

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ESTIMATES OF SHEAR MODULUS IN THE EARTH'S OUTER AND INNER CORES

In [13], based on results of the interpretation of precursor waves propagating in the lower part of the Earth's outer core [12], we suggest that the shear modulus therein should differ from zero and reach $2 \cdot 10^{12}$ Pa for oscillations of approximately 1 Hz. This conclusion is based on the following fact: the velocity gradient of a longitudinal wave increases in the lower part of the outer core. In addition, it is assumed that the bulk shear modulus (particularly, in the core) is a monotonic function of depth.

Let us estimate the maximum and minimum μ values in the core based on the following three conditions: (i) the shear modulus is a negative function; (ii) the adiabatic modulus of the bulk shear k_S in the core is monotonic; and (iii) the adiabatic modulus of the bulk shear is a continuous function of pressure (in our case, depth). The first condition is evident. The second and third conditions are quite justified from the physical point of view, because the outer and inner cores allegedly have similar compositions based on iron compounds. We can calculate the shear modulus based on formulas for the seismic parameter Φ and the velocity of transverse waves v_S:

$$\Phi(z) = v_P^2 - \frac{4}{3}v_S^2 = \frac{k_S}{\rho}, \quad v_S = \sqrt{\frac{\mu}{\rho}}, \qquad (1)$$

$$\mu = \frac{3}{4}(\rho v_P^2 - k_S) \geq 0.$$

The last formula includes density parameter ρ, which can be found from the Williamson–Adams equations. This parameter is a function of the shear modulus μ. Therefore, determination of the upper and lower boundaries of the shear modulus μ requires checking the density distribution in both the core and mantle. Formula (1) implies that the minimum value corresponds to the maximum k_S value and vice versa. In order to find the maximum and minimum values of the shear modulus, we can determine reliably three points in the $k_S(z)$ curve for the mantle. The first point corresponds to the core–mantle interface. The second point lies at the F zone–inner core interface. The third point corresponds to the core center. We can pass a quadratic parabola across the three points, and the parabola will satisfy the requirements mentioned above.

We shall determine the maximum k_S value in the following way. Based on specified values of the function of the velocity of longitudinal waves in the Earth's core $v_P(z)$ and density $\rho(z)$ [13], we shall draw curve $k_S(z)$ for the case when μ is equal to zero in the entire core. This curve yields k_S values at two points: at the core–mantle and zone F–inner core interfaces. Let us determine the k_S value at the third point, i.e., the core center, assuming that the Poisson coefficient σ = 0.44. This value is determined from the standard model and is actually much higher than that for the solid (crystalline) iron core. Then, we shall choose a quadratic parabola, which would pass across the three points: $k_{P2900} = 6.471 \cdot 10^{11}$, $k_{P5000} = 1.164 \cdot 10^{12}$, and $k_{P6371} = 1.325 \cdot 10^{12}$ Pa. The formula for such a parabola is as follows:

$$k_{max} = -6.20919 \cdot 10^{11} + 5.48821 \cdot 10^8 z - 38200.8 z^2.$$

At the same time, the formula meets the requirements listed above and is limited from the top by the boundary curve.

Minimum values for k_S are determined from the condition of the zero μ value at the core–mantle interface and μ values in the outer core corresponding to the Poisson coefficient σ = 0.28. This condition corresponds to the solid crystalline state of the iron core. For the minimum k_S values in this case, $k_{P2900} = 6.471 \cdot 10^{11}$, $k_{P5197} = 9.113 \cdot 10^{11}$, and $k_{P6371} = 9.543 \cdot 10^{12}$ Pa. The parabola equation will be as follows:

$$k_{min} = -2.06844 \cdot 10^{10} + 2.95534 \cdot 10^8 z - 22366.7 z^2.$$

Estimation curves k_{max} and k_{min}, as well as the corresponding distributions of density and shear modulus, are presented in Figs. 1 and 2.

ESTIMATES OF VISCOSITY OF THE EARTH'S CORE

We shall consider material in the Earth's core as a viscous liquid. For the motion of particles under the forcing, we can adopt [14] from the Navier–Stokes equation

$$\frac{\partial^2 \dot{u}}{\partial x^2} = \frac{\rho}{\eta}\frac{\partial \dot{u}}{\partial t}, \qquad (2)$$

where u is the displacement of particles of the medium; $\dot{u} = \frac{\partial u}{\partial t}$; η is the dynamic viscosity; t is the time; and x is the space coordinate.

Integrating the equation by t, we obtain

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{\eta}\frac{\partial u}{\partial t} + \text{const}.$$

Accepting that the velocity of particle displacement in the medium is equal to zero at the initial moment $t = 0$, we obtain const = 0.

In addition, since the viscous liquid behaves as a solid body at high frequencies, we can write the wave equation [15] for transverse oscillations as

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{\mu}\frac{\partial^2 u}{\partial t^2}, \qquad (3)$$

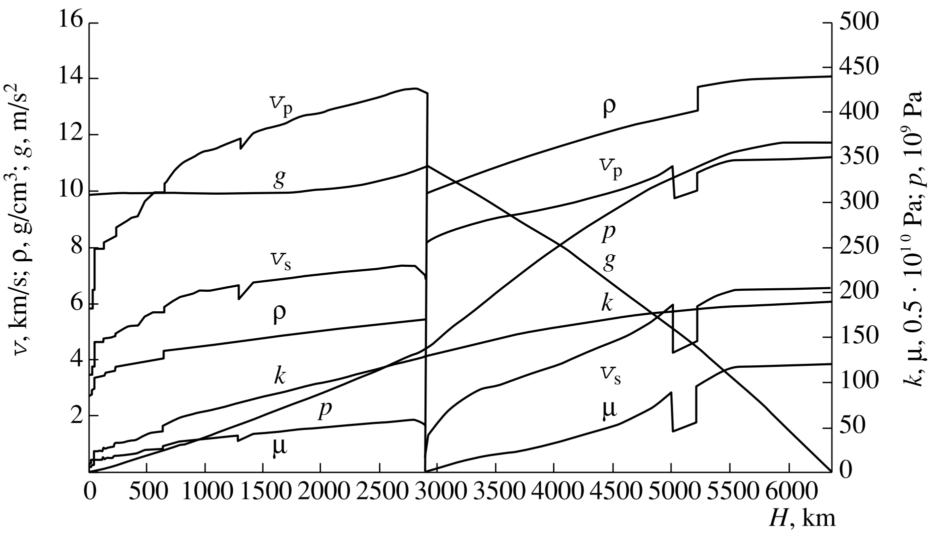

Fig. 1. Distribution of minimum k and maximum μ values in the Earth's core.

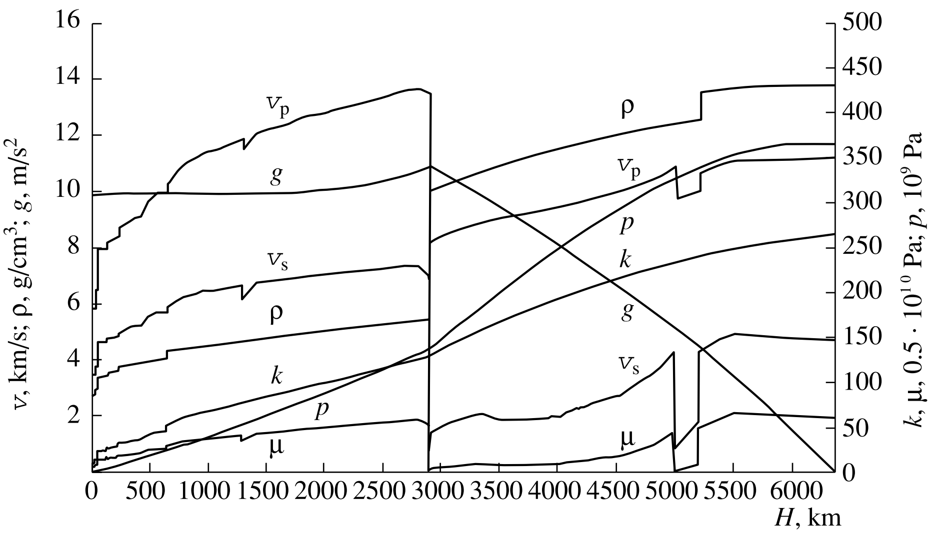

Fig. 2. Distribution of maximum k and minimum μ values in the Earth's core.

which is valid for the elastic isotropic medium.

Equating the right-hand parts of Eqs. (2) and (3), we obtain

$$\frac{\rho}{\mu}\frac{\partial^2 u}{\partial t^2} = \frac{\rho}{\eta}\frac{\partial u}{\partial t}. \quad (4)$$

Integrating (4) by t, we obtain

$$\frac{\partial u}{\partial t} = \frac{\mu}{\eta}u + \text{const}. \quad (5)$$

Assuming that the velocity of particle displacement in the medium is equal to zero at the initial moment $t = 0$, we obtain const = 0. Then, Eq. (5) can be written as

$$\mu = \eta\frac{\partial \ln u}{\partial t}.$$

Thus, the shear modulus of the viscous liquid is equal to its viscosity coefficient multiplied by the velocity of variation in the natural logarithm of the displacement of medium particles.

Let us consider the harmonic oscillations of the medium. In this case, $u = u_0e^{-i\omega t}$, and Eq. (5) is written as

$$\mu = -i\omega\eta,$$

where ω is the angular frequency and i is the imaginary unit.

Taking the modulus of the last expression, we obtain

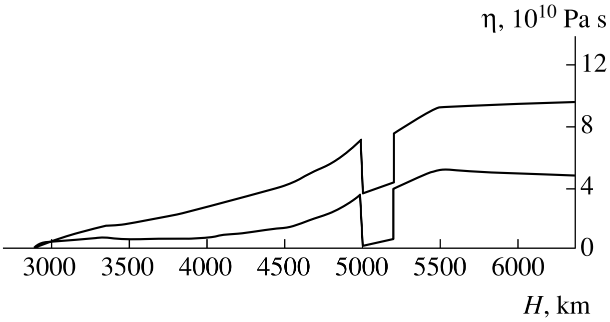

Fig. 3. Distribution of maximum and minimum η values in the Earth's core.

$$\mu = \omega\eta.$$

Assuming $\tau = \frac{1}{\omega}$, where τ is the relaxation time, we get the known equation linking the viscosity coefficient and shear modulus of the viscous liquid [15]

$$\eta = \tau\mu.$$

Now we can estimate viscosity in the Earth's core. Accepting $\omega = 2\pi$ rad/s, which corresponds to the oscillation frequency of seismic waves equal to 1 Hz [12] or $\tau = \frac{1}{2\pi}$ s, we get the distribution of minimum and maximum η values presented in Fig. 3.

Assume that $v = \dot{u}$. Then, Eq. (2) is written as

$$\frac{\partial^2 v}{\partial x^2} = \frac{\rho}{\eta}\frac{\partial v}{\partial t}. \qquad (6)$$

Solution of Eq. (6) is given by the formula [14]

$$v = v_0 e^{-x/\delta} e^{i(x/\delta - \omega t)},$$

where $\delta = \sqrt{\frac{2\eta}{\rho\omega}}$.

Thus, the amplitude of the seismic wave will decrease by e times over the distance δ. In addition, both longitudinal and transverse wave zones in the upper part of the outer core incorporate a shadow zone linked with a significant velocity reduction in the outer core, relative to the velocity in the lower mantle. Moreover, the zone can be much wider for the transverse waves, relative to the longitudinal ones. Therefore, the probability of the detection of transverse waves in seismograms is negligible.

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