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Approximation of Seismic Travel Time Curve with Convex Splines

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Despite the present, wide spread notion that the Earth's crust and mantle are essentially inhomogeneous media the idea is still widespread that there exist velocity characteristics of these media. This idea is based on seismic data within the framework of the so-called one-dimensional models of the media (i.e., of the media in which the velocity of propagation of elastic waves is a function of only one coordinate, usually the Earth's radius or the depth). The methods of determining the seismic wave velocity profile from travel time curves for one-dimensional models of the media, are developed fairly completely and they are mainly based on the formulas of conversion of the travel-time curves [1, 2]. However, for every experimental travel-time curve it is certainly not possible to determine the velocity function from the conversion formulas. For this it is necessary that the travel-time curve possesses certain properties, in particular, the property of convexity. In the present article, an approximate spline-function is constructed, by reducing the arbitrary experimental travel-time curve to a form suitable for the application of the conversion formulas.

§1. TRAVEL-TIME CURVE OF SEISMIC WAVE PROPAGATING IN VERTICALLY-INHOMOGENEOUS MEDIA AND CONVEX CUBIC SPLINES

We write the equation of a seismic travel-time curve in the form

$$x(H)=2\int_0^H \frac{p\,dz}{\sqrt{n^2(z)-p^2}},\qquad t(H)=2\int_0^H \frac{n^2(z)\,dz}{\sqrt{n^2(z)-p^2}}.$$

In these equations the parameter p is equal to the derivative of the travel-time curve at the point x(H), i.e., $p = dt/dx(H)$ and $n(z) = 1/v(z)$; x(H) corresponds to the point of emergence of a seismic ray (buried at depth H) to the Earth's surface. For the refracted waves, $dt/dx(H) = 1/v(H)$ and the continuity of the travel-time curve is conserved only for a continuous monotonic increase of velocity with depth. Consequently, for the travel-time curve, the inequality $dt/dx(z) > dt/dx(H)$ must be satisfied for all $z < H$ (the condition of monotonicity of the first derivative of the travel-time curve). This condition can be written in terms of the second derivative, as $d^2t/dx^2 < 0$. These arguments also hold for the cuspidal loop. In this case, we should only take into consideration the fact that the beginning of the cuspidal branch corresponds to large values of x and t. For travel-time curves of reflected waves the inequality $d^2t/dx^2 > 0$ holds. Thus, in the solution of inverse kinematic problems in seismics for axially symmetrical homogeneous media, the experimental travel-time curve should satisfy the condition of convexity: upward, if the travel-time curve relates to the refracted waves; downward, if it relates to reflected waves.

However, the experimental travel-time curves practically never satisfy these requirements. First of all, this is due to the real media being inhomogeneous not only in depth but also in the horizontal directions. Therefore, the travel-time curves of the seismic waves, propagating in the real media, cannot satisfy the convexity condition. Furthermore, every experimental travel-time curve is distorted by random errors connected with the separation of definite wave types and with the variation in the observed quantities, for instance, time of arrival of seismic waves and epicentral distance. Even in the case of a horizontally homogeneous medium, the travel-time curve will not be convex. Therefore, before determining the velocity function v(z) from the conversion formulas, we should smooth the experimental travel-time curve $t = \hat{t}(x)$ by means of some convex (concave) curve T(x), which in the given norm, deviates least from the experimental travel-time curve.

We shall now construct the smoothing function with the aid of cubic splines. By cubic spline we mean a function which is fused from different segments of a cubic parabola. The cubic spline is continuous and has continuous first and second derivatives, while the third derivative can have a discontinuity with a finite jump at the points of connection [3, 4]. An approximate representation of the functions with the aid of splines has a definite advantage over the representation by polynomial functions. One of the deficiences of the polynomials, in this connection, is the fact that their behavior in the neighborhood of some point governs their behavior as a whole. Splines are free from this deficiency. Besides, they possess such important properties as the properties of best approximation, minimum curvature or minimum norm [3].

The conversion formulas of the seismic travel-time curve enable us to determine the maximum depth of the seismic ray, to which is ascribed the definite propagation velocity of seismic waves. This velocity, as was already mentioned, is determined as a reciprocal of the derivative of the travel-time curve at the relevant point on this curve. The problem of differentiating the experimental function $\hat{t} = \hat{t}(x)$ is an ill-posed problem. The special difficulty here is finding the derivatives at the initial and extremal points of the travel-time curve, which play a primary role in the determination of the parameters of the individual layers of the medium. Splines enable us to solve this problem fairly effectively.

We now take up the problem of smoothing. Let the source of the elastic waves be located at some point on the x-axis, these waves propagate in the lower plane of the elastic medium with a one-dimensional law of variation of velocity v(z). At the points of observation x_i (i = 1, 2, ... n) on the x-axis, the values of $\hat{t}_i = \hat{t}(x_i)$ of the travel time curve of the seismic wave $t = t(x)$, are determined. We assume that the values of x_i are known exactly and that instead of the values of t_i, their approximations $\hat{t}_i = \hat{t}(x_i)$ having deviations from the true values, $\delta t_i = t_i - \hat{t}_i$, are known. Such an assumption is valid, since the errors in the determination of x_i can be reduced to the errors in the determination of t_i. In order to convince ourselves of this, it is enough to expand $t(x)$ in the neighborhood of x_i in Taylor series. Then we have

$$t(x)=t(x_i)+t'(x_i)\cdot(x-x_i)+\dots .$$

If instead of x, we substitute $x_i + \delta x_i$, we have

$$t(x_i+\delta x_i)=t(x_i)+t'(x_i)\delta x_i+\dots=t_i+\delta t_i=\tilde{t}_i.$$

Here $\delta t_i = t'(x_i)\delta x_i + \dots$. In this $\tilde{t}(x)$ is not a convex function. We have to construct a cubic spline, i) which is convex upwards, from the approximate values of the travel-time curve $\hat{t}_i$, given in the grid

$$\Delta: a=x_0<x_1<\dots<x_N=b,$$

if we consider the travel-time curve of refracted waves, and convex downwards, if we consider the travel-time curve of reflected waves, and ii) which is the best approximation of the experimental travel-time curve in Euclidean norm.

We assume that such a spline $T(x)$ exists. Then, writing the system of linear equations, governing the second derivatives of the interpolated spline for itself in terms of the values of the spline at the interpolation grids, we have [3]

$$h_{i-1}T''_{i-1}+2(h_{i-1}+h_i)T_i''+h_iT''_{i+1}=6\left(\frac{T_{i+1}-T_i}{h_i}-\frac{T_i-T_{i-1}}{h_{i-1}}\right), \quad (1)$$

where $i = 1, 2, \dots, N-1$; $h_i = x_{i+1}-x_i$ and T_i'' is the second derivative of the spline at the point x_i. By virtue of the convexity condition of the spline, we have $T_i'' < 0$ for the refracted waves and $T_i'' > 0$ for the reflected waves. We introduce the notations

$$\lambda_i=\frac{h_{i-1}}{h_i+h_{i-1}} \text{ and } f_i=\frac{h_ih_{i-1}}{6}[\lambda_iT''_{i-1}+2T_i''+(1-\lambda_i)T''_{i+1}].$$

In terms of these we write the system of linear equations for determining the spline at the grids, as

$$(1-\lambda_i)T_{i-1}-T_i+\lambda_iT_{i+1}=f_i. \quad (2)$$

In order that this system is uniquely solvable, it is necessary to assign conditions, governing the two free parameters. We set $T_0 = \hat{t}_0$, $T_N = \hat{t}_N$. Then, the solution of the system (2) can be written as

$$T_i=\sum_{j=1}^{N-1}\mu_{ij}f_i-\mu_{i1}(1-\lambda_1)\hat{t}_0-\mu_{i,N-1}\lambda_{N-1}\hat{t}_N, \quad (3)$$

where μ_{ij} are elements of the inverse matrix for the system (2)

Our problem will be solved, if we can construct such a spline which would guarantee a minimum of the functional

$$S=\sum_{i=1}^{N-1}p_i(T_i-\hat{t}_i)^2 \quad (4)$$

under the condition $T_i'' < 0(T_i'' > 0)$. In other words, as a solution of the system (1), we take values of T_i'', realizing a minimum of the functional (4) under the conditions $T_i'' < 0$ in the case of the travel-time curve of refracted waves and $T_i'' > 0$ in the case of the travel-time curve of reflected waves. Here $p_i(p_i > 0)$ are given numbers.

§2. BOUNDARY CONDITIONS FOR THE APPROXIMATING SPLINE

The initial system of linear equations (1) determines N-1 unknown parameters in T_i''. At the same time, the interpolated cubic spline is determined by N + 1 parameters. Thus, for solving the problem it is necessary to extend the definition of the two free parameters. Usually these parameters are determined by the boundary conditions, imposed on the interpolated spline at the extremities. We shall examine the boundary conditions for the approximation of the seismic travel-time curve.

Since the second derivatives of the travel-time curve are not usually known at any point on the travel-time curve, at the extremities of this curve, or at the initial and limiting points we can assume that the second derivatives vanish, i.e., $T_0'' = T_N'' = 0$. We may note that if the velocity in the medium varies according to a linear law, then at the point $x_0 = 0$, $T_0'' \equiv 0$. Obviously, we can also set $T_0'' = T_1''$ and $T_N'' = T_{N-1}''$. If $x_0 = 0$, we can make use of the information on the first derivative of the travel-time curve at the origin. For the reflected waves, as we know already, $t'(0) = 0$. For the refracted waves $t'(0) = 1/v(0)$, and $v(0)$ is the velocity on the surface, which can be determined from other observations. Using the expression for $t'(x)$, we have [3]: $t'(0)=\frac{\hat{t}_1-\hat{t}_0}{h_0}-\frac{h_0}{6}T_1''-\frac{h_0}{3}T_0''$. From this we have

$$T_0''=3\frac{\hat{t}_1-\hat{t}_0}{h_0^2}-\frac{3}{h_0}t'(0)-\frac{1}{2}T_1''.$$

For the continuous seismic observations, when the observation points are situated fairly close to one another, we can use some approximation of the derivatives at the extremities of the segment of observations. For instance [5]

$$t'(x_0)=t(x_0,\ x_1)-h_0t(x,\ x_1,\ x_2)+h_0(h_0+h_1)\times t(x_0,\ x_1,\ x_2,\ x_3)+r_0'.$$

$$t'(x_N)=t(x_N,\ x_{N-1})+h_{N-1}t(x_N,\ x_{N-1},\ x_{N-2})+$$
$$+h_{N-1}(h_{N-1}+h_{N-2})t(x_N,\ x_{N-1},\ x_{N-2},\ x_{N-3})+r_N',$$

where $t(x_i, x_{i+1}, \dots x_{i+j})$ is the divided difference of the (j-i)-th order; r_0', r_N' are the remaining terms which are usually discarded.

§3. DETERMINATION OF THE ELEMENTS OF THE INVERSE MATRIX

We write the system (2) in the matrix form

$$LT=F. \tag{5}$$

Here L is the three-diagonal matrix of the system (2); $T=\{T_i\}$ is the unknown vector; $F=\{f_i\}$ is the given vector. The solution of the equation (5) is written as $T = MF$, where $M = L^{-1}$. It can be easily seen that the elements of the matrix L depend only on the position of the grid points of the net Δ and hence the elements of the matrix M also depend only on the net and not on the values of the ordinates at the grid points.

To determine the elements of the matrix M, we write the system of linear equations as

$$\begin{cases} -\mu_{i1}+(1-\lambda_2)\mu_{i2}=0, \\ \lambda_2\mu_{i1}-\mu_{i2}+(1-\lambda_3)\mu_{i3}=0, \\ \quad\dots \\ \lambda_{i-1}\mu_{ii-1}-\mu_{ii}+(1-\lambda_{i+1})\mu_{ii+1}=1, \\ \quad\dots \\ \lambda_{N-2}\mu_{iN-2}-\mu_{iN-1} \qquad =0, \end{cases} \tag{6}$$

$$i=1,\dots,N-1.$$

This system is obtained from the condition $ML = E$, where E is a unit matrix. We will solve the system (6) by the method described in [3]. Then, we have

$$q_k=\frac{1-\lambda_{k+1}}{1-\lambda_{k-1}q_{k-1}}, \qquad q_0=0, \qquad \lambda_0=0,$$

$$u_k=\frac{\lambda_{k-1}u_{k-1}-\delta_{ki}}{1-\lambda_{k-1}q_{k-1}}, \qquad \delta_{ki}=\begin{cases}0 & \text{when } k\neq i\\ 1 & \text{when } k=i\end{cases},$$

$$\mu_{iN-1}=u_{N-1}, \quad \mu_{ik}=q_k\mu_{ik+1}+u_k, \quad k=1,\dots,N-1.$$

Thus, varying the subscript i from 1 to N-1, we obtain all the values of the elements of the inverse matrix M. Here, if the net is uniform, i.e., $\lambda_k = 0.5$, the procedure of determining the elements of M becomes definitely stable in the sense that the errors of determinations attenuate, since $q_k=\frac{k}{k+1}<1$.

§4. MINIMIZATION OF THE FUNCTIONAL S(T").

We write the function (3) in the expanded form

$$S(T'')=\sum_{i=1}^{N-1}\left\{\sum_{j=1}^{N-1}\mu_{ij}f_j-\mu_{i1}(1-\lambda_1)\bar{t}_0-\mu_{iN-1}\lambda_{N-1}\bar{t}_N-\bar{t}_i\right\}^2=$$

$$=\sum_{i=1}^{N-1}\left\{\frac{1}{6}\sum_{j=1}^{N-1}\mu_{ij}h_jh_{j-1}[\lambda_jT''_{j-1}+2T_j''+(1-\lambda_j)T''_{j+1}]-\right.$$

$$\left.-\mu_{i1}(1-\lambda_j)\bar{t}_0-\mu_{iN-1}\lambda_{N-1}\bar{t}_N-\bar{t}_i\right\}^2=$$

$$=\sum_{i=1}^{N-1}\left\{\frac{1}{6}\sum_{j=1}^{N-1}[\dot{\mu}_{ij+1}\lambda_{j+1}+2\dot{\mu}_{ij}+\dot{\mu}_{ij-1}(1-\lambda_{j-1})]T_j''-\right.$$

$$\left.-\mu_{i1}(1-\lambda_1)\bar{t}_0-\mu_{iN-1}\lambda_{N-1}\bar{t}_N-\bar{t}_i\right\}^2.$$

Here $\mu_{ij}^*=\mu_{ij}h_jh_{j-1}$ and $\mu_{i0}^*=\mu_{iN}^*=0$, $\lambda_N=\lambda_0=0$, $T_0''=T_N''=0$.

The functional S(T") is convex in respect of T_i'', bounded below and continuous on the convex set $\mathcal{T}=\{T''|T_i''\leq 0\}$. Consequently, S(T") has a single minimum in respect of T".

We make the substitution $-g_i^2 = T_i''$, if we consider the travel-time curve of the refracted waves, and $g_i^2 = T_i''$ in the case of reflected waves. Then, from the minimization of functional (4) in the convex set $\mathcal{T}$ we pass to the minimization without restraints. The functional S can be written in the form

$$S(g)=\sum_{i=1}^{N-1}\left\{\text{sign}\sum_{j=1}^{N-1}\alpha_{ij}g_j-t_i^*\right\}^2, \tag{7}$$

where

$$\text{sign}=\begin{cases}-1 & \text{if} \quad T_i''<0\\ 0 & \text{if} \quad T_i''=0,\\ 1 & \text{if} \quad T_i''>0\end{cases}$$

$$\alpha_{ij}=\frac{1}{6}[\dot{\mu}_{ij+1}\lambda_{j+1}+2\dot{\mu}_{ij}+\dot{\mu}_{ij-1}(1-\lambda_{j-1})],$$

$$t_i^*=\mu_{i1}(1-\lambda_1)\bar{t}_0+\mu_{iN-1}\lambda_{N-1}\cdot\bar{t}_N+\bar{t}_i.$$

$$i,\ j=1,\ 2,\dots,\ N-1.$$

If we choose boundary conditions $T_0'' = T_1''$ and $T_N'' = T_{N-1}''$, then we should set

$$\alpha_{i1}=\frac{1}{6}[\dot{\mu}_{i-1}(2+\lambda_1)+\dot{\mu}_{i2}\lambda_2],$$

$$\alpha_{iN-1}=\frac{1}{6}\{\dot{\mu}_{iN-2}(1-\lambda_{N-2})+\dot{\mu}_{iN-1}(3-\lambda_{N-1})\}.$$

If the first derivative of the travel-time curve is known at the origin $t_0' = 1/v_0$ ($t_0' = 0$), then in this case we have

$$\bar{t}_i^*=\frac{1}{2}\mu_{i1}^*\lambda_1\left(\frac{1}{v_0h_0}-\frac{\bar{t}_1-\bar{t}_0}{h_0^2}\right)+\mu_{i1}(1-\lambda_1)\bar{t}_0+\mu_{iN-1}\lambda_{N-1}\bar{t}_N+\bar{t}_i,$$

$$\alpha_{i1}=\frac{1}{6}\left[\mu_{i1}\left(2-\frac{\lambda_1}{2}\right)+\mu_{i2}\lambda_2\right].$$

For the minimization of the functional (7) we make use of one of the methods of descent. We shall minimize the functional (7) by the method of steepest descent [6]. In conformity with the scheme of the method, we construct the minimizing sequence

$$g^{k+1}=g^k-\beta_k\,\text{grad}\,[S(g^k)],$$

where

$$\text{grad}[S(g)]=\left\{\frac{\partial S}{\partial g_1},\ \frac{\partial S}{\partial g_2},\dots,\ \frac{\partial S}{\partial g_{N-1}}\right\},$$

$$\frac{\partial S}{\partial g_e}=4g_e\,\text{sign}\sum_{i=1}^{N-1}p_i\left\{\left[\text{sign}\sum_{i=1}^{N-1}\alpha_{ij}g_i^2-\bar{t}_i^*\right]\alpha_{ie}\right\}.$$

We choose β_k from the conditions: Let us verify β_k^0.
If $S(g^k-\beta_k^0\,\text{grad}\,[S(g^k)])$ decreases, we take $\beta_k^1=2\beta_k^0$, and so on.
If $S(g^k-\beta_k^0\,\text{grad}\,[S(g^k)])\geqslant S(g^k)$, we divide the step into two, and so on.

The flow diagram of the program of computing β_k, according to the above scheme, is given in [7]. As a zero approximation for $(g_i^0)^2$ we can take, for example, the second derivative of the quadratic parabola, approximating the experimental travel-time curve

$$T_2 = a_0 + a_1(x-x_0) + a_2(x-x_0)^2,$$
$$(g_i^0)^2 = T_2'' = 2a_2, \qquad i=1,\ 2,\ \ldots,\ N-1.$$

We continue the iteration procedure till the difference $S_{k+1} - S_k$ becomes less than some preassigned quantity ε. ε is chosen empirically, depending on the required accuracy of computations and computation time.

As the weighting factors p_i, it is advisable to choose quantities (as shown by numerical experiments) which are proportional to the second derivatives of the unknown spline. Thus, at the (k +1)-th the step of searching the minimum of the functional (7), we should set

$$p_i^{k+1} = \frac{1}{2}\left\{ (g_i^k)^2 \Big/ \sum_{j=0}^{N} (g_j^k)^2 + p_i^k \right\}, \quad p_i^0 = 1/(N+1).$$

§ 5. CONSTRUCTION OF THE SPLINE T(x)

As soon as the minimum of the functional S is found, i.e., after the relevant values of $T_i'' = -g_i^2$ ($T_i'' = g_i^2$) are found, without any difficulty whatsoever we can determine T_i of the unknown spline T(x) at the grid points of the net Δ from the travel-time curves (3). After determining T_i, we can write the unknown spline function, which approximates the experimental travel-time curve, as follows [3]:

$$T(x) = \frac{1}{6h_i}\{[(x-x_i)^3 - h_i^2(x-x_i)]T''_{i+1} - [(x-x_{i+1})^3 - h_i^2(x-x_{i+1})]T_i''\} +$$
$$+ \frac{1}{h_i}[(x-x_i)T_{i+1} - (x-x_{i+1})T_i],$$
$$x \in [x_i,\ x_{i+1}]; \qquad i=1,\ 2,\ \ldots,\ N-1.$$

We shall show that the spline so obtained is actually convex. We write the second derivative of the spline at an arbitrary point of the segment $[x_{i+1},\ x_i]$. By virtue of the linearity of the second derivative T''(x) =

$= \frac{1}{h_i}[T''(x_i)(x_{i+1}-x) + T''(x_{i+1})(x-x_i)]$. Since $T''(x_i) < 0$ $(T''(x_i) > 0)$ and $(x_{i+1}-x)/h_i \geq 0$, $(x-x_i)/h_i \geq 0$, $T''(x) < 0$ $(T''(x) > 0)$ for all the values of $x \in [x_{i+1},\ x_i]$ (i = 1, 2, ..., N-1). This means that the function is convex.

Table.

N	x	T		V*		d^2t/dx^2	
		experimental	smoothed	before smoothing	after smoothing	before smoothing	after smoothing
1	0,0	0,00	0,00	3,56	5,41	0,0000	0,0000
2	1,0	0,27	0,18	4,04	5,41	-0,0674	-0,0001
3	6,0	1,09	1,11	6,05	5,42	0,0345	-0,0001
4	11,5	2,23	2,12	5,10	5,44	-0,0243	-0,0001
5	17,1	3,13	3,15	6,43	5,45	0,0089	-0,0001
6	22,6	4,09	4,16	5,32	5,46	0,0028	-0,0001
7	28,0	5,09	5,14	5,81	5,48	-0,0086	-0,0001
8	33,0	5,99	6,06	4,61	5,49	0,0266	-0,0001
9	38,0	7,16	6,97	4,97	5,50	-0,0329	-0,0001
10	43,4	8,00	7,95	6,54	5,51	0,0151	-0,0001
11	49,4	9,08	9,04	5,30	5,52	0,0031	0,0001
12	51,2	9,94	9,91	5,97	5,53	-0,0058	-0,0001
13	61,6	11,12	11,24	6,07	5,54	0,0051	-0,0001
14	65,2	11,76	11,89	5,14	5,55	0,0115	-0,0001
15	7[illegible],7	12,87	12,88	5,42	5,56	-0,0152	-0,0001
16	76,2	13,73	13,87	7,06	5,57	-0,0004	-0,0001
17	81,7	14,63	14,86	4,79	5,58	0,0249	-0,0001
18	87,2	15,84	15,84	5,75	5,60	-0,0376	-0,0001
19	92,7	16,57	16,82	6,50	5,61	0,0303	-0,0001
20	98,2	17,70	17,80	4,44	5,64	-0,0043	-0,0002
21	103,7	18,90	18,78	4,64	5,67	0,0006	-0,0002
22	109,2	20,05	19,74	5,13	5,72	-0,0081	-0,0003
23	114,6	20,90	20,68	9,57	5,78	-0,0253	-0,0004
24	120,0	21,42	21,61	6,75	5,85	0,0415	-0,0005
25	125,4	22,52	22,53	4,94	5,95	-0,0214	-0,0006
26	130,7	23,43	23,41	6,00	6,07	0,0078	-0,0007
27	136,0	24,40	24,27	5,13	6,24	0,0029	-0,0010
28	142,4	25,65	25,28	5,36	6,47	-0,0056	-0,0007
29	145,1	26,11	25,69	6,87	6,53	-0,0249	-0,0004
30	147,8	26,44	26,10	9,15	6,59	-0,0019	-0,0006
31	153,2	27,04	26,91	8,31	6,78	0,0059	-0,0009
32	158,4	27,70	27,66	7,96	7,10	-0,0037	-0,0017
33	169,2	29,00	29,10	7,70	7,96	0,0045	-0,0012
34	174,6	29,65	29,76	11,22	8,25	-0,0196	-0,0004
35	180,0	30,07	30,41	9,27	8,49	0,0265	-0,0004
36	185,5	30,89	31,06	6,31	8,56	-0,0081	-0,0004
37	195,8	32,21	32,24	9,14	8,85	-0,0014	-0,0003
38	201,2	32,80	32,85	8,86	8,96	0,0027	-0,0002
39	211,8	34,04	34,02	9,01	9,12	-0,0030	-0,0001
40	217,1	34,63	34,60	9,71	9,15	0,0000	0,0000

The first derivative of the spline is determined from the formulas [3]

$$T'(x)=\frac{1}{2}\left\{\left[\frac{h_i}{3}-\frac{(x_{i+1}-x)^2}{h_i}\right]T_i''\right.$$

$$\left.-\left[\frac{h_i}{3}-\frac{(x-x_i)^2}{h_i}\right]T_{i+1}''\right\}+\frac{T_{i+1}-T_i}{h_i}.$$

The apparent velocities are equal to the reciprocals of the first derivatives of the spline-function T(x): $v^*(x) = 1/T'(x)$.

§6. EXAMPLE OF REALIZATION OF THE ALGORITHM

The above algorithm of approximating a travel-time curve by a convex spline-function is realized as a program in the algorithmic language ALGOL-60 for the system ALGOL-BESM 6. As an example to illustrate this, we shall give the results of approximation, by means of a convex spline-function, of the travel-time curve of the first incidences, obtained at an observation point in the experimental investigations, by DSS method. In the table are set out the data concerning the parameters of the travel-time curve (x, t, v^*, d^2t/dx^2) before and after the approximation. The root-mean-square of the deviations of the approximations for these data, is $\delta \approx 0.16$ sec.

The suggested method of approximation of individual travel-time curves enables us to apply effectively the conversion formulas to the solution of the inverse kinematic problem in seismics. The cubic splines appear to be most effective apparatus for solving this problem. Actually, if we make use of splines of higher order, we cannot guarantee the convexity of the approximating function in the entire domain of approximation. This fact prevents us from using the known algorithms (for purposes of approximating individual travel-time curves) of smoothing the functions of a single variable, by cubic splines, since in these algorithms no constraints are imposed on the sign of the second derivative of the spline [4].

The above algorithm of approximating the travel-time curve is valid under the assumption of a plane-parallel distribution of the seismic wave velocities with depth. However, at the early stage it can also be applied to the travel-time curves, obtained for spherically-symmetric media. For this it is enough if we use the equations of the travel-time curve on the sphere [8].

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