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Numerical Solution to the Problem of Optimum Seismic Network Design over the Earth's Surface

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Abstract—Based on a nonstatistical approach (the C-optimum test) to the problem of design of experiments, a multicriteria objective (MO) function is formulated to account for diverse natural and antropogenic factors, such as the location of teleseismic stations, the Earth's surface relief, and detection conditions at observation points (the gain factor of instruments). The numerical solution to the problem of the teleseismic network optimization is found by minimization of the MO function.

INTRODUCTION

The creation of an optimum observation network allows us not only to effectively solve a variety of problems, but also to save expenses by minimizing the number of necessary observation points. This variety includes the localization of earthquake hypocenters, seismicity monitoring over the entire Earth's surface, study of the internal constitution of the Earth, etc.

Recall that the optimum observation network in the problem of determining hypocenter coordinates and origin times of earthquakes means a grid of measurements providing specified errors in the desired parameters for the least number of seismic stations. A set of observation points in the theory of design is termed the experimental design. The optimum designs must obey certain criteria depending on the problem to be solved experimentally. The problem of determining the optimum seismological network belongs to a class of problems on searching for discrete optimum designs. In general, the coordinates of observation points in the optimum network are determined using numerical algorithms.

This paper continues the previous one [Burmin, 1994], where the basic principles are formulated for constructing the optimum teleseismic network on the Earth's surface. Here, based on a nonstatistical approach (the C-optimum criterion), a multicriteria objective (MO) functional is introduced to allow for the position of seismic stations in the modern network, the Earth's surface relief, and detection conditions of seismic waves at observation sites. Then, the coordinates of the optimum network points on the Earth's surface are determined. According the C-optimum criterion, a design is optimal when the quantities $\|K\| \cdot \|K^+\|$ or $\|K^+\|$, where K is the design matrix, $\|\circ\|$ is any norm, and K^+ is the generalized reciprocal matrix, take on the least values. The minimization of the MO functional faces a number of difficulties. The most severe of them is that such a functional depends on a great number of parameters, some of which are unknown, e.g., the distribution of microseisms (seismic noise) over the Earth's surface. For this reason, the problem was simplified to develop an efficient minimization algorithm and to obtain a result close to the optimum one.

CONSTRUCTION OF THE MULTICRITERIA OBJECTIVE FUNCTIONAL

(1) We use the following system of linear algebraic equations for earthquake hypocenter parameters on a sphere [Burmin, 1994]

$$\begin{aligned} &Uu_i + Vv_i + Ww_i - Qq_i \\ &= \cos(d_i/R_i) - \tau_i\tau_0 c_i^2/(r_0 R_i), \end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, n$; n is the number of observation points; $U = X_0/r_0$; $V = Y_0/r_0$; $W = Z_0/r_0$; $Q = T_0/r_0$; $u_i = x_i/R_i$; $v_i = y_i/R_i$, $w_i = z_i/R_i$; $q_i = \tau_i c_i^2/R_i$, $r_0 = R_E - h$; X_0, Y_0, and Z_0 are the hypocenter coordinates; $T_0 \approx \tau_0$ is the earthquake origin time; x_i, y_i, and z_i are the coordinates of seismic stations; $R_0 = \sqrt{X_0^2 + Y_0^2 + Z_0^2}$, $R_i = \sqrt{x_i^2 + y_i^2 + z_i^2}$; R_E is the Earth's radius; τ_i is the arrival time of a seismic wave to the station; c_i are the effective velocities of seismic waves, which are given by the ratio of the hypocentral distance d_i (along straight lines) to the time of travelling along their rays; and h is the hypocenter depth. For $\tau_0 = 0$, τ_i are the times of propagation of seismic waves from their source to recording stations.

In matrix form, (1) becomes

$$K\mathbf{p} = \mathbf{f}, \tag{2}$$

where $\mathbf{p} = \{p_j\}$ is the vector of desired parameters; $j = 1, 2, 3$, and 4; K are the matrices of the system of linear equations; $\mathbf{f} = \{f_i\}$ is the vector of observables; and $i = 1, 2, \ldots, n$.

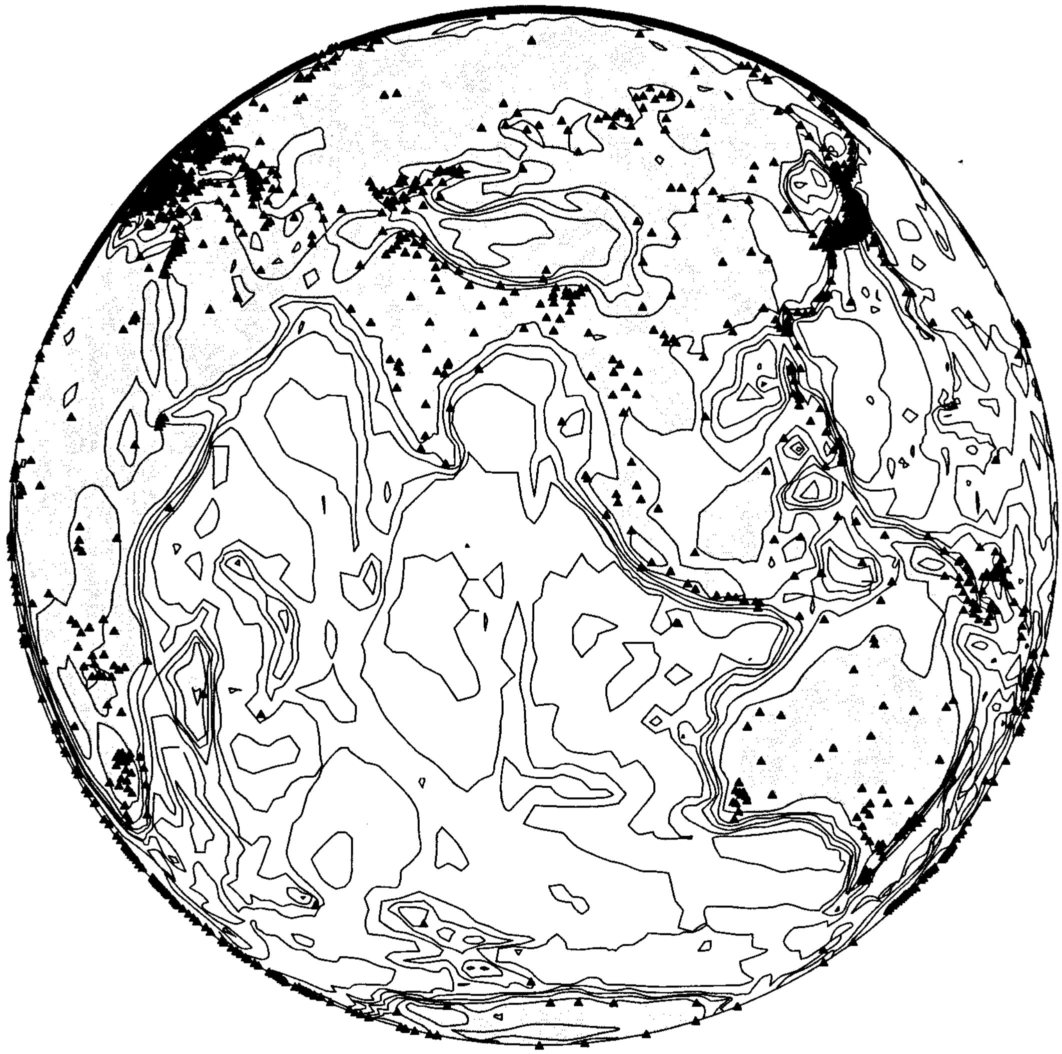

Fig. 1. Existing network of seismic stations on the eastern hemisphere of the Earth.

The estimated error in the determined parameters p_j is of the form [Burmin, 1994]

$$||\Delta p|| \leq ||K^+|| \cdot ||\sin(d/R)\rho/\alpha|| \cdot |\Delta\tau|, \quad (3)$$

where Δp is the error vector of the desired parameters; $K^+ = (K^TK)^{-1}K^T$; T denotes the conjugation operation; $d = \{d_i\}$; $R = \{R_i\}$; $\rho = \{\rho_i\}$ are the weighting factors accounting for both measurements of varying accuracy at seismic stations and systematic deviations in the determined τ_i, related to the inhomogeneity of real media; $\alpha = \{\alpha_i\}$ is the derivative of the travel time curve at corresponding point $\alpha_i = t_i'$; $\Delta\tau$ is the maximum error in the determined τ_i; and henceforth, $||\circ||$ signifies the Euclidean norm.

(2) By varying the coordinates of observation points for a given n, it is possible to find their positions in which (3) takes on the least value. Since the right side of inequality (3) is the estimate of the maximum error in the determined hypocenter parameters, we come to the minimax problem on the determination of the optimum arrangement of seismic stations on the Earth's surface. Thus, the optimum observation network must minimize the function

$$J_0 = ||K^+|| \cdot ||\sin(d/R)\rho/\alpha||. \quad (4)$$

(3) If an earthquake were detected at any point on the Earth's surface, the optimum observation network to localize earthquake hypocenters could be obtained by placing seismic stations at the corners of a regular tetrahedron inscribed into the Earth's sphere [Burmin, 1994]. Yet, since earthquakes of various magnitudes are detected at different distances from their hypocenters, the station spacing in an observation network must be chosen in accordance with the minimum intensity of earthquakes to be detected. The minimum intensity detected at a certain distance depends on the gain factor of the measuring instrument, which, in turn, depends on

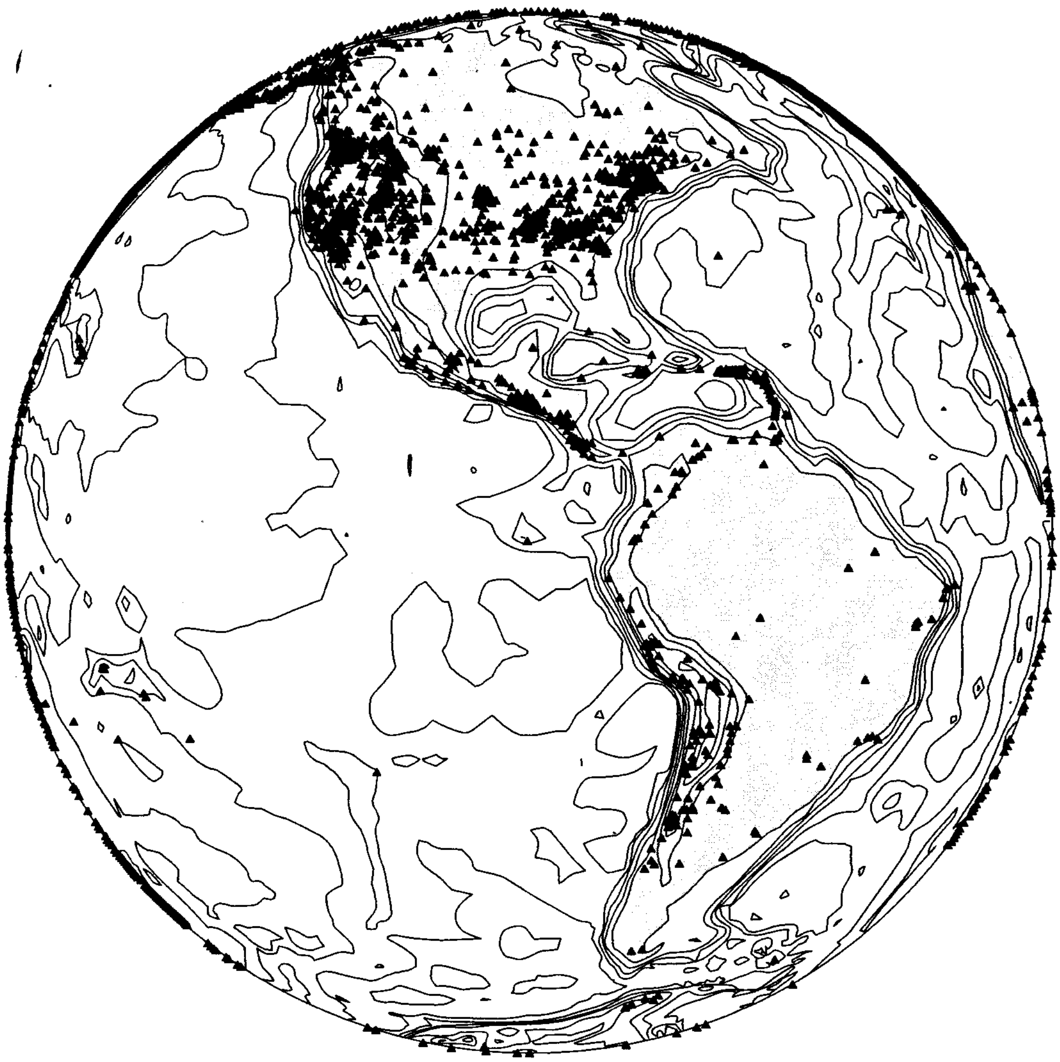

Fig. 2. Existing network of seismic stations on the western hemisphere of the Earth.

the microseism level (seismic noise) at the detection point and is defined as the ratio between the microseism oscillation amplitude on a seismic record, reduced to the 1-mm level of the visible record, and the ground oscillation amplitude expressed in millimeters. In designing a seismological network, it is natural to specify some threshold of the gain factor, below which the amplification is not allowed. Consequently, the installation sites of seismic stations must be chosen so that the observed microseism level is lower than *a priori* given value. For the minimum noise at the observation site, the gain factor is maximum. The optimum conditions for detecting will be observed if the function

$$J_1 = \left(\sum_{i=1}^{n} A_i^2\right)^{1/2}, \quad (5)$$

takes on the minimum value provided that $|A_i| \le A_0$, where A_0 is the noise threshold.

Burmin [1995] showed that, if the dimensions of the design area substantially exceed the sizes of the detection zone of earthquakes with given minimum magnitudes, the local optimum network is a coverage of the area by regular hexagons (the so-called hexagonal lattice). Thus, for small given earthquake magnitudes, the problem of the optimum seismic network is reduced to the coverage of the entire Earth's surface by the maximum number of regular spherical hexagons with a given edge. Burmin [1994] obtained an optimum (in the sense of the minimization of functional (4)) network, in which the observation points are at the corners of polyhedrons, where (except 12 corners of the original icosahedron) six equilateral triangles converge, and the five isosceles triangles converge at the 12 corners. Such an observation geometry will detect, without omissions, earthquakes with a magnitude of four for the gain factor of 40000 and earthquakes with a magnitude of three for the gain factor of 200000. It is clear that the given observation system is determined ambiguously, with

Fig. 3. Calculated optimum seismic net.

accuracy to an arbitrary rotation about any of three arbitrarily chosen and mutually orthogonal axes passing through the center of the terrestrial globe.

(4) It is economically feasible to choose the optimum observation geometry in a manner that the greatest number of possible points of the system be coincident with, or at least close to, the existing observation points. In relation to this, we shall minimize the function of an integer argument

$$J_2 = 1/m, \qquad (6)$$

where m is the number of pairs of coinciding points ($m \geq 1$), i.e., the designed and existing observation network points, the spacing of which is not greater than a specified value ε. This value is picked by considering the scale of the designed network.

(5) Because of a considerable variability of the Earth's surface relief, it is desirable to place seismic stations at the sites most accessible to observations. This imposes limitations on the heights (depths) of observation sites. In general, we must approach the minimum of the function

$$J_3 = \left[\sum_{i=1}^{n} Z_i^2\right]^{1/2}, \qquad (7)$$

provided that the heights (depths) of the observation points satisfy the inequality

$$|Z_i| \leq Z_0, \quad i = 1, \ldots n, \qquad (8)$$

where Z_i is the height (depth) of the ith station relative to sea level and Z_0 is a given positive number.

Thus, the problem of concern here is reduced to the multicriteria optimization in which the optimum observation points minimize the objective functional

$$\Theta = \Theta(J_0, J_1, J_2, J_3). \qquad (9)$$

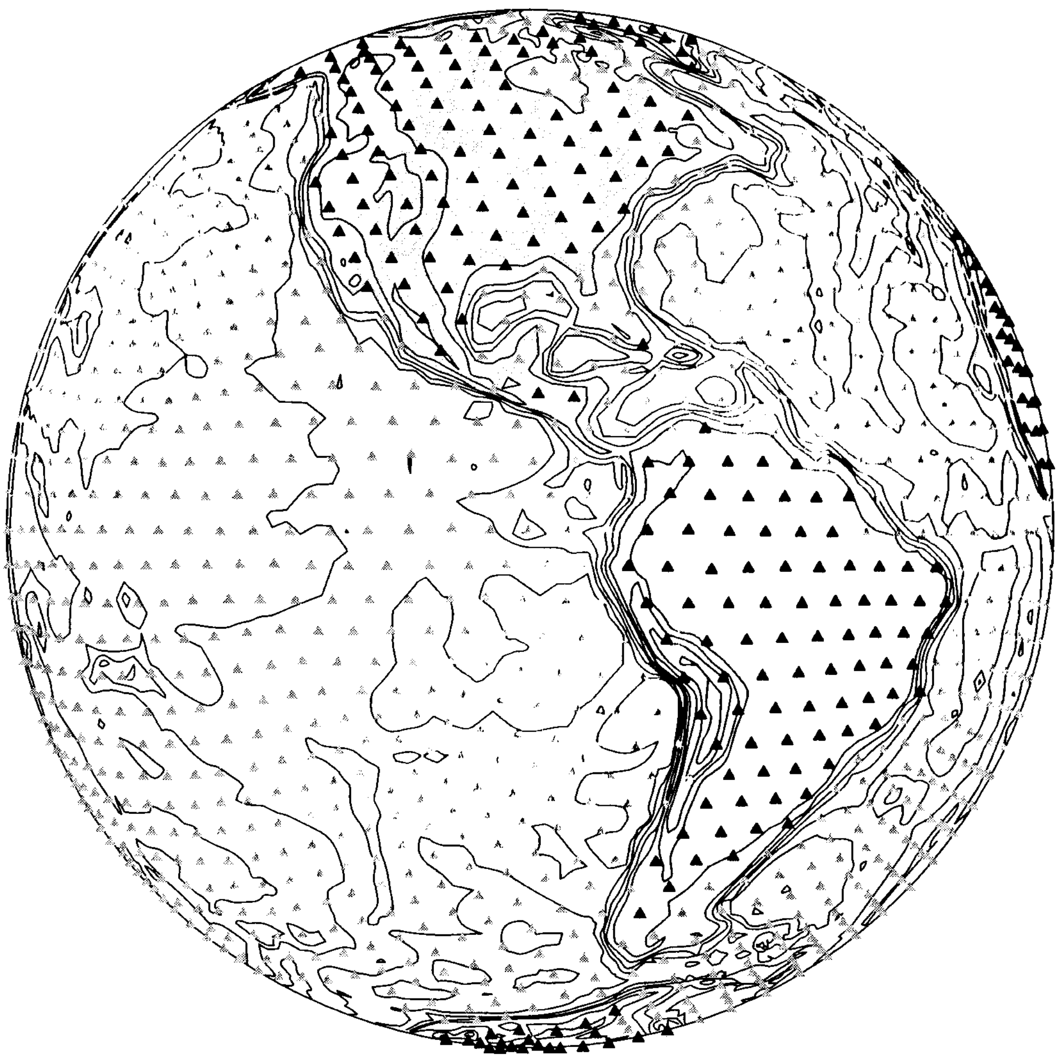

Fig. 4. Calculated optimum seismic net.

NUMERICAL SOLUTION BY A METHOD OF WEIGHTING SUMS WITH A POINTWISE ESTIMATION OF WEIGHTS

The problem of the multicriteria optimization is solved by the minimization of the objective functional Θ. An efficient approach used in similar problems is the one of weighting sums with a pointwise estimation of the weights [Shtoier, 1992]. In this approach, functional J_0 and the criterion functions J_i $(i = 1, 2, 3)$ are multiplied by strictly positive scalar quantities called the weights ω_i $(i = 0, 1, 2, 3)$. $\omega^T = \{\omega_i\}$ are the weighting vectors that are normalized so that the sum of the components of each vector is one. Then, all the weighting functions are added up, and the problem is reduced to the minimization of the objective functional in which the components of the weighting vectors play the role of significance for each individual criterion function involved in the objective functional.

In that case, the objective functional takes the form

$$\Theta = \{\omega^T J\} = \omega_0 J_0 + \omega_1 J_1 + \omega_2 J_2 + \omega_3 J_3, \quad (10)$$

and the problem is to simultaneously minimize (10) on a set of the weighting vectors.

This approach to finding the optimum design is the most rigorous. However, while it is not possible to obtain an explicit expression for the objective functional Θ in the problem of interest, it is difficult to choose such an optimization method that would allow us to simultaneously minimize all the criterion functions, because the individual functions entering Θ are not unimodal.

It is therefore appropriate to simplify our problem so that, the efficiency of the minimization algorithm is sufficient and the result will be close to the optimum solution.

For this, we use the optimum quasihomogeneous configuration of observations obtained by Burmin

[1994]. When searching for the minimum of the objective functional, this configuration is considered a rigid framework and will only be rotated in the required directions. This means that the criterion functional J_0 value will not affect the minimum of the objective functional Θ. Furthermore, we have no information on the microseism intensity distribution over the Earth's surface and therefore cannot consider the detection conditions at observation sites; consequently, the noise factor will be neglected for the moment. Moreover, we abandon the condition for the function (7) minimum, but retain the constraints for the heights (depths) of observation points. It is clear that condition (8) cannot hold true for all observation points, since vast areas on the Earth exist (mountain systems), where the average altitudes are relatively high. In view of the above, solving problem (7) under condition (8) is replaced by the following problem

$$N(|Z_j| \geq Z_0) \longrightarrow \min, \quad (11)$$

which implies that the observation system must have the minimum number of the observation points whose heights (depths) exceed specified values.

Thus, vector ω takes the form $\omega^T = \{0, 0, 1, 0\}$, and functional Θ is written as

$$\Theta = J_2. \quad (12)$$

In order to find the minimum of function (12), we consider the following procedure. Applying a net with the optimum set of observation points over the existing network of seismic stations, we vary the configuration of the former relative to the latter in a manner that each point of the net runs all the values in a domain Ω that is cut by the solid angle θ on the Earth's surface. Angle θ is taken from the following considerations. Each of the existing seismic stations falls into one of the spherical triangles given by the divisions of the faces of a spherical icosahedron. Each of the triangles can be circumscribed by a circle with a radius $r \approx L/\sqrt{3}$, whose center is at the center of the triangle. The circle cuts a cone in a sphere of radius R, so that the cone angle is $\chi = 2\arcsin(r/R)$. By varying increments in the polar coordinates φ and λ within the ranges $\delta\varphi \subset [0, \chi]$ and $\delta\lambda \subset [0, \chi]$, we cover the entire surface of the Earth. For each of the fixed positions of the optimum net with respect to the existing network, we determine distances d_i satisfying the condition $d_i \leq \varepsilon$ and find the number m of the "coinciding" points. In the course of searching for the J_2 minimum, points in domain Ω were selected discretely, at a step of ε. The ε value was chosen to be 1/3 of the distance between the nearest points of the optimum net (about 15 km). The Z_0 value was set at 6000 m, which mostly corresponds to the maximum depths and heights of the Earth.

Figures 1 and 2 show the existing network of seismic stations on the eastern and western hemispheres of the Earth. A total of 4466 stations are plotted in Figs. 1 and 2. The optimum net contains 2562 observation points. The results of the calculations are presented in Figs. 3 and 4. The number of coinciding stations was found to be 133.

CONCLUSION

An approximate solution to the problem on the optimum array of seismic stations over the Earth's surface was obtained. This allows us to detect, without omission, earthquakes with a magnitude of at least 4.5 under the condition that the instrument gain factor is 40000. The optimum net includes 2562 stations, whereas the existing global network, whose observation sites are located only on the ground and are rather unevenly distributed, contains about 4500 stations. Our derivation takes into account existing stations and the location of practically inaccessible sites. It is quite obvious that the optimum net is profitable economically and provides the acquisition of more complete information.

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