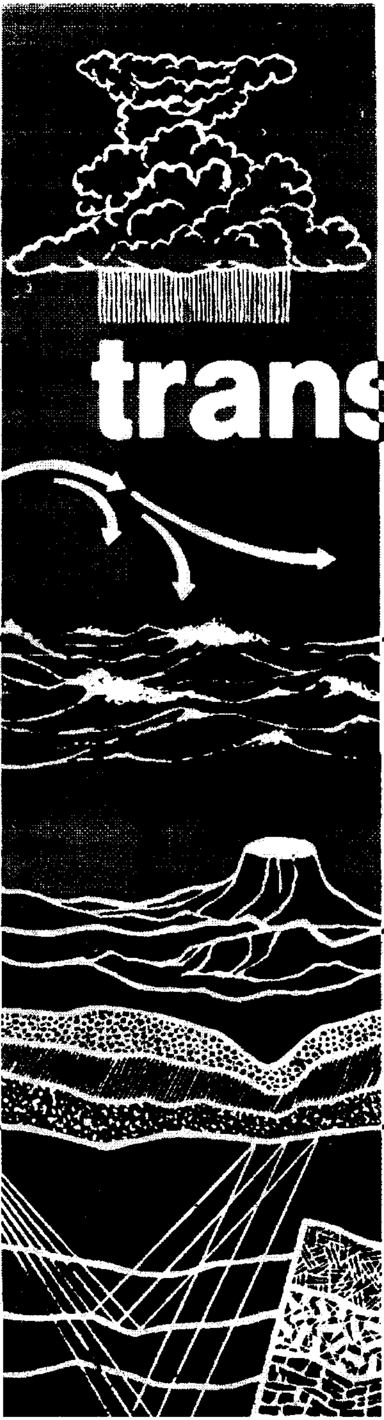
VOLUME 340
NUMBER 1 1996

ISSN 1083-3552



transactions

(DOKLADY)

OF THE

Russian Academy of Sciences

earth science sections

WILEY

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AMERICAN GEOLOGICAL INSTITUTE
and the
AMERICAN GEOPHYSICAL UNION

UDC 550.344

Optimal Geometry of a Teleseismic Observation Network[1]

V.Yu. Burmin

Shmidt Joint Institute of Physics of the Earth,
Russian Academy of Sciences, Moscow
(Presented by Academician V.N. Strakhov, July 13, 1993)

In terms of the theory of experimental design, the existing teleseismic observation network is non-optical and is inadequate for many purposes, such as locating the hypocenters of weak ($M \leq 4.5$) earthquakes, monitoring seismicity over the entire surface of the globe, investigating the interior structure of the earth, and the like. The creation of an optimum array not only would allow the above tasks to be performed more effectively, but would also save large amounts of money by minimizing the number of sites.

In this paper we present a system of linear algebraic equations that relate the coordinates of earthquake hypocenters to the coordinates of seismic stations on the surface of the terrestrial ellipsoid. This system is then used to derive estimates of the error in determining hypocenter characteristics, after which we consider the optimum placement of measurement sites over the surface of the globe. Finally, we formulate the basic principles for constructing an optimum teleseismic observation system.

1. In a geocentric frame of reference, the relationship between the coordinates of the hypocenters and of the seismic stations can be described by a system of nonlinear equations

$$(X_0 - x_i)^2 + (Y_0 - y_i)^2 + (Z_0 - z_i)^2 = r_i^2, \quad (1)$$

where X_0, Y_0, Z_0 are the hypocenter coordinates, x_i, y_i, z_i are the coordinates of the seismic stations, and $\mathbf{r}_i = \mathbf{R}_i - \mathbf{R}_0$, where

$$|\mathbf{R}_i| = R_i = \sqrt{x_i^2 + y_i^2 + z_i^2},\ |\mathbf{R}_0| = R_0 = \sqrt{X_0^2 + Y_0^2 + Z_0^2},$$

$$i = 1, 2, \ldots, n.$$

Expanding the expressions in parentheses in Eq. (1) and applying the equations for R_i and R_0, we obtain

$$X_0 x_i + Y_0 y_i + Z_0 z_i = 0.5(R_0^2 + R_i^2 - r_i^2). \quad (2)$$

[1]Translated from: Optimal'naya geometriya seti teleseysmicheskikh nablyudeniy. Doklady Rossiyskoy Akademii Nauk, 1994, Vol. 334, No. 3, pp. 364-367.

The quantities r_i^2 are given by $r_i^2 = R_0^2 + R_i^2 - 2R_0R_i \cos \psi_i$, where $\psi_i = d_i/R_i$ is the angle between the vectors $\mathbf{R}_0$ and $\mathbf{R}_i$, and the d_i are the epicentral distances. Let us write $r_0 = R_0$. Substituting ψ_1 into the equations for r_i^2 and substituting the r_i^2 into Eq. (2), then dividing the right and left sides by r_0R_i, we obtain

$$Uu_i + Vv_i + Ww_i = \cos(d_i/R_i) = \cos(\psi_i), \tag{3}$$

where $U = X_0/r_0$, $V = Y_0/r_0$, $W = Z_0/r_0$, $u_i = x_i/R_i$, $v_i = y_i/R_i$, $w_i = z_i/R_i$, $r_0 = R_E - h$, h is the depth of the hypocenter from the ground surface, and R_E is the earth's radius at the point in question.

System (3) defines the three unknowns, x_0, y_0, and z_0. But in attempts to locate distant earthquakes, it is most desirable to determine four hypocentral parameters, X_0, Y_0, Z_0 and τ_0, the hypocentral coordinates and the time of the quake. For this purpose we can use the system

$$Uu_i + Vv_i + Ww_i - Qq_i = \cos(d_i/R_i) - \tau_0\tau_i c_i^2/(r_0R_i), \tag{4}$$

where $Q = T_0/r_0$, $q_i = \tau_i c_i^2/R_i$, $c_i = R_i(\tau_i - \tau_0)$.

2. Let us now rewrite systems (3) and (4) in matrix form:

$$Kp = f, \tag{5}$$

where $p = \{p_j\}$ are the unknown parameters, $j = 1, 2, 3$ or $j = 1, 2, 3, 4$; K denotes the matrices of the systems, and $f = \{f_i\}$ are the measurements; $i = 1, \ldots, n$.

If both the vector of the free terms f and the matrix K are specified with errors $\Delta f \neq 0$ and $\Delta K \neq 0$, then the error of vector p is given [1] by

$$\bar{K}\Delta p = \Delta f - \Delta Kp. \tag{6}$$

The solution of this equation is

$$\Delta p = \bar{K}^{+}(\Delta f - \Delta Kp).$$

The error Δp of the vector is defined by the condition

$$||\Delta p|| \leq ||\bar{K}^{+}||\ ||\Delta f - \Delta Kp||. \tag{7}$$

Let us return to system of linear equations (3). We shall assume that the errors in the elements of matrix K and in the right sides of the equations result only from errors in the arrival times τ_i of the waves, whose absolute values may be assumed to be $|\delta\tau_i| = \rho_i|\Delta\tau|$. The weighting factor ρ_i expresses both the numbers of measurements at the stations and the systematic error resulting from the discrepancy between the model and the true velocities. Then, $\Delta K = 0$, and $\Delta f \neq 0$, and the elements of vector Δf are $\delta f_i = -\sin(d_i/R_i)\delta d_i$. If we assume that $\delta d_i = \delta\tau_i/\alpha_i$, where $\alpha_i = t_i$ is the derivative of the time-distance plot at the point in question, we can write

$$\delta f_i = -\sin(d_i/R_i)\delta\tau_i/\alpha_i \quad (\alpha_i \neq 0).$$

The full error vector for Eq. (3) is then subject to the condition

$$||\Delta p|| \leq ||\tilde{K}^{+}|| \, ||\sin(d/R)\rho/\alpha|| \, |\Delta\tau|. \qquad (8)$$

Suppose that system of linear equations (5) applies to the case in which not only X_0, Y_0, and Z_0, but also T_0, are unknown, with $\Delta K \neq 0$ and $\Delta f \neq 0$. Then,

$$\delta f_i = -\sin(d_i/R_i)\delta\tau_i/\alpha_i - \delta\tau_i\tau_0 c_i^2/(r_0 R_i),$$

$$(\Delta K p)_i = -\delta\tau_i T_0 c_i^2/(r_0 R_i).$$

If we assume $\tau_0 \approx T_0$, we obtain

$$(\Delta f - \Delta K p)_i = -\sin(d_i/R_i)\delta\tau_i/\alpha_i.$$

Thus, in this case too we arrive at Eq. (8), and $p_4 = T_0/r_0$.

Condition (8), a majorant, gives the assured maximum error in determining the hypocenter parameters of distant earthquakes. According to the criterion of C-optimality [2], if we vary the coordinates of the observation sites, we may choose a position such that estimate (8) is minimized. Since the right side of inequality (8) is an estimate of the maximum error in determining the hypocenter parameters, we arrive at a minimax problem of determining the optimum placement of seismic stations on the globe. Thus, the problem of determining the optimum geometry of the measurement system may be treated as a problem of minimizing the target function

$$J = ||\tilde{K}^{+}|| \, ||\sin(d/R)\rho/\alpha)||,$$

or, more precisely, some functional of target function J [1]. The principal contribution to J is made by the function $J_1 = \|\tilde{K}^{+}\|$.

Consider the function J_1. System (5) has the most stable solution when, for a given norm of the matrix of the initial system, the determinant of the normal system of equations has its maximum value [2].

Consider again system (3). The Binet-Cauchy formula for the determinant of the matrix of the normal equations for this system is

$$\det(K^{\tau}K) = \sum_{i_1<i_2<i_3}^{n} \begin{vmatrix} u_{i_1} & v_{i_1} & w_{i_1} \\ u_{i_2} & v_{i_2} & w_{i_2} \\ u_{i_3} & v_{i_3} & w_{i_3} \end{vmatrix}^2 . \qquad (9)$$

The value of each determinant under the summation sign in (9) is 6 times the volume of a tetetrahedron with vertices $M_{i_1}(u_{i_1}, v_{i_1}, w_{i_1})$, $M_{i_2}(u_{i_2}, v_{i_2}, w_{i_2})$, $M_{i_3}(u_{i_3}, v_{i_3}, w_{i_3})$, located on a unit sphere, with one vertex at the center of sphere, and is independent of the positions of the earthquake hypocenters. Of course, if all of the vertices of the tetetrahedron other than that at the center of the sphere lie on the same great-circle arc, then the volumes of the corresponding tetrahedra will be zero. This means that if all measurement sites are on the same great-circle arc,

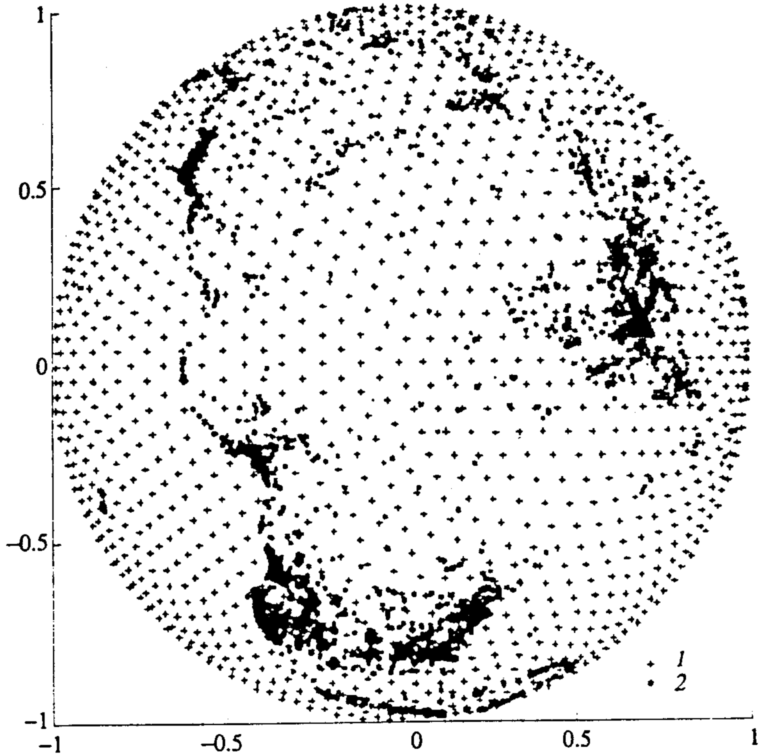

Fig. 1. Projection of unit hemisphere bearing measurement points onto plane. Center of projection coincides with a vertex of the regular eicosahedron: *1*) Points on optimum teleseismic grid; *2*) points of existing detection system.

the determinant for the system of normal equations will vanish and the system will be degenerate. But determinant (9) has its maximum value when the sum of the squares of the volumes of all tetrahedra inscribed in the unit sphere is a maximum. On the other hand, each set of points on the sphere is a set of vertices of some polhyedron, whose volume is equal to the sum of the volumes of the corresponding polyhedra with one common vertex at the center of the sphere.

Similar reasoning applies to system (4).

We know [3] that the polyhedron with the greatest volume among all polyhedra with a given number of vertices that are located on some particular smooth, convex surface must be a true polyhedron with triangular faces, that is, a polyhedron that has only triangular faces, which lie in different planes. In particular, the volume of a polyhedron inscribed in the unit sphere is subject to the inequality [3]

$$V \le \frac{2k}{3}\cos^2\frac{\pi f}{2k}\cot\frac{\pi e}{2k}\left(1-\cot^2\frac{\pi f}{2k}\cot^2\frac{\pi e}{2k}\right),$$

where e, f and k are the number of vertices, the number of faces, and the number of edges of the convex polyhedron. The inequality becomes an equality in the case of a regular polyhedron with triangular faces. There exist three such polyhedra: the tetrahedron, the octahedron, and the eicosahedron.

Because earthquakes of different magnitudes are detectable at different hypocentral distances, the distances between stations in the observation network should be chosen so as to detect the smallest-magnitude earthquake that we desire to observe. Antonova et al. [4] suggest that m_{min} be determined from the formula

$$m_{min} = \log(\gamma A_n/VT) + \delta(\Delta, h, s(T, \omega)). \quad (10)$$

In this formula, γ is the minimum possible signal-to-noise ratio at which the signal can be extracted (in practice, it is assumed that $\gamma \sim 1.5$); A_n is the amplitude of the noise on the seismic trace, in mm; and V is the instrument gain, 10^3 (the gain of seismic stations is usually chosen so that A_n is of the order of 1 mm); T is the wave period; and $\delta(\Delta, h, s(T, \omega))$ is the "calibration function," which has been studied rather thoroughly. The values of the calibration function for various types of instruments and various parts of the former USSR are given in a standardization document [5].

Thus, the points of an optimum observation network are the vertices of a regular or semiregular polyhedron which: a) has triangular faces; b) has edge lengths appropriate to the smallest earthquake magnitudes that need to be detected, so that an earthquake is recorded by at least 3 (or 4) stations. To construct such a system, we use an eicosahedron inscribed within the terrestrial sphere. The eicosahedron has the largest number of vertices of all regular polyhedra with triangular faces. The edge length of an eicosahedron inscribed within the terrestrial sphere is $l \approx 7076$ km. According to Eq. (10), in such a system, earthquakes with a magnitude of about 5 will be detected by three stations. To obtain a system with smaller minimum base lines, we subdivide the spherical triangles into four parts, as follows: we draw lines between the midpoints of all sides of each triangle among the arcs of a great circle, thus subdividing each triangle into four triangles with sides of about 3538 km. Continuing this subdivision procedure, we obtain a detection system with even smaller base lines. In general, with an n-fold subdivision of the spherical triangle generated by the eicosahedron, we obtain $e_n = 10 \times 4^n + 2$ vertices. The fourfold subdivision yields a subdivision with base lines of about 442 km and with 2562 measurement sites. The projection of a hemisphere bearing such a system onto a plane is shown in Fig. 1. If the number of instruments were increased to 40,000, such a system would detect all earthquakes with a minimum magnitude of about 4; with 200,000 stations, it would detect all quakes with a minimum magnitude of about 3.5. If we treat the earth as a sphere, the points in each system are the vertices of a polyhedron; all vertices except the 12 belonging to the original eicosahedron are points of convergence of 6 equilateral triangles, and the remaining 12 are points of convergence of 5 isosceles triangles. If we ignore the ellipticity of the earth, the polyhedron constructed in this way will have faces that are not equilateral triangles; but it will nonetheless remain a true polyhedron.

Our procedure has not, of course, specified some single system: all systems obtained by an arbitrary rotation around any of three arbitrarily chosen mutually perpendicular axes passing through the center of the sphere satisfy the conditions. It is advisable to choose the system in such a way that most of the points coincide with or at least are close to the existing measurement sites.

The optimum teleseismic observation network is a covering of the terrestrial sphere with triangles at whose vertices the measurement sites are located. Over nearly the entire surface of the globe these triangles form hexagons having measurement sites at their vertices and centers; in 12 regions, the triangles form pentagons whose centers coincide with the vertices of the original eicosahedron. The optimum seismologic array on a surface is a covering of that surface by such hexagons, with measurement sites located at their vertices and centers. Thus, the optimum seismologic observation network on the globe and on the plane coincide for almost the entire earth, with the exception of 12 regions. The size of these regions depends on the number of subdivisions of the original eicosahedron. If the original regular polyhedron is an octahedron or tetrahedron, there will be respectively 6 and 4 such regions, but the triangles within them will be grouped into squares and triangles respectively.

In designing optimum systems, it is of course desirable that the national and regional networks fit into the global network.

Most of the measurement sites (about 1600) of an optimum global seismologic network would be located at sea. The importance of making measurements on the sea floor as well as on land is indisputable; the technical difficulties involved in long-term ocean measurements are not insuperable. Other geophysical measurements could be combined with the seismologic measurements at these sites.

REFERENCES

1. Burmin, V.Yu. Izv. Akad. Nauk SSSR, Fizika Zemli, No. 5, pp. 34-42, 1986.
2. Burmin, V.Yu. Izv. Akad. Nauk SSSR, Tekhnicheskaya kibernetika, No. 2, pp. 195-200, 1976.
3. Feyesh, Tot L. Raspolozheniye na ploskosti, na sfere i v prostranstve (Arrangements on a Plane, on a Sphere, and in Space), Fizmatgiz, Moscow, 1958 (appears to be a translation).
4. Antonova, L.V., Z.I. Aronovich and N.V. Kondorskaya. In: Magnituda i energeticheskaya klassifikatsiya zemletryaseniy (Magnitudes and Energy Classifications of Earthquakes), Moscow, 2, pp. 195-202, 1974.
5. Instruktsiya o poryadke proizvodstva i obrabotki nablyudeniy na seysmicheskikh stantsiyakh YeSSN SSSR (Making of Measurements and Processing of Data at Seismic Stations of the USSR Uniform Seismic Measurement Systems), Nauka Press, Moscow, 1981.