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# Optimal Placement of Seismic Stations for Registration of Near Earthquakes

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The problem of optimal placement of seismic stations for various initial data is studied in nonstatistical terms. Solutions are given for determination of the optimal hypocenter and the optimal geometry of the observation system for an infinite planning region.

## DETERMINATION OF THE COORDINATES OF HYPOCENTERS OF NEAR EARTHQUAKES AND A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

From practical experience, as well as numeric calculations, we know that the accuracy in determining the parameters of earthquake hypocenters largely depends on the reciprocal positions of the seismic stations and their positions relative to the hypocentral zone.

The issue of optimal placement of a regional network of seismic stations was first studied by Vvedenskaya in 1955 [1]. She investigated the relationship between the errors in the determination of epicentral coordinates and the azimuthal range of stations when the earthquake epicenters are determined by intersections. This problem was later investigated by other authors [2 - 8], who used various alternative approaches. The most rigorous analysis of the determination of an optimal position of stations in a regional network was probably made in [3, 6], where this problem was analyzed in terms of the present-day statistical theory of experiment planning.

The theory of experiment planning, based on mathematical statistics, has studied exhaustively the so-called linear continuous normalized plans, i.e., observations systems for which the relationship between observed values $w_i$ $(i=1, 2, \ldots, n)$ and the sought-for parameters $p_j$ $(j=1, 2, \ldots, m)$ is expressed by a linear equations system

$$Kp = w, \qquad (1)$$

where $w^T=\{w_i\}$; $p^T=\{p_j\}$; $K$ is the matrix of a system representing the mathematical model of the relationship, and the ratio of the number of measurements at each observation point to the total number of measurements in the observation system may take any value from 0 to 1 (for a more detailed discussion of normalized plans, see, e.g., [9]).

The statistical evaluation theory estimates the optimal linear values of the unknown parameters $p$ by the least-squares method:

$$\hat{p}=(K^TK)^{-1}K^Tw$$

(where the superscript $T$ indicates transposition). A covariance matrix of such estimates, if the observations are conducted at different points with variances $\sigma_{\hat{w}}^2$, appears as

$$D(\hat{p})=\sigma_w^2(K^TK)^{-1}.$$

The efficacy of the experiment is judged by the magnitude of error in the evaluation of the parameters $p$. The parameters $p$ are vector quantities, and generally the accuracy of the estimates $\hat{p}$ is characterized by all elements of the variance matrix $D(\hat{p})$. The various estimates of $p$ can therefore be compared in different ways. Depending on the methods of comparing the estimates, the experiment planning theory considers various plan optimality criteria. The popular criteria are $A$-, $E$-, and $D$-optimality [9].

$A$-optimality is satisfied by plans with a minimal mean variance of parameter estimates, or the least value of the trace of the covariance matrix $D(\hat{p})$. $E$-optimal plan corresponds to the least maximum eigenvalue of the covariance matrix. The plans optimal by $D$-test have the smallest determinant of the covariance matrix among alternative plans.

A system of seismic stations for earthquake registration is a typical example of a discrete plan with unit measurement at each point. Continuous plans can only be considered a good approximation of discrete plans if a large number of measurements are made at each observation point. In our case, when the number of measurements at a point is equal to 1, we cannot use directly the results derived from continuous plans. Besides, the traditional experiment planning theory assumes that the matrix of the initial linear equations system is defined exactly, and the measurement errors of $w_i$ are random. In a seismic experiment, the requirement that no disturbance be introduced into the matrix $K$ is not always met, and the errors of the initial data can be either random or systematic. Systematic errors may be generated, for example, by some unknown structural elements existing in the region.

In [10], a nonstatistical method of experiment planning was suggested, where the optimality of a discrete plan could be probed regardless of the type of errors (whether systematic or random) in the initial data and allowing for perturbations in the plan matrix $K$. According to this criterion, a plan is optimal if it minimizes the degree to which linear equations system (1) is conditioned, or the norm of the matrix inverse to the plan matrix. Basically, plans optimal by the nonstatistical test minimize, on the set of plans, the maximum error of the parameter $p$. In [10], it was shown that, with an appropriate choice of the norm for the matrix $K$, the nonstatistical test is equivalent to $A$-, $E$-, and $D$-optimality, i.e., it was demonstrated that statistical and nonstatistical approaches to experiment planning are not contradictory.

This paper considers, in the framework of the nonstatistical approach, the problem of optimal placement of seismic stations with various initial data.

## 1. System of Linear Equations Linking the Coordinates of Near Earthquake Foci and the Coordinates of the Recording Stations

Near earthquakes are usually defined as earthquakes separated from the recording stations by a distance of a few hundred kilometers.

We will select a Cartesian coordinate system such that the plane $XY$ lies on the Earth surface and the axis $Z$ is directed downward. We will define near earthquakes as the earthquakes whose coordinates $X$, $Y$, $H$ satisfy with a given accuracy the relation

$$(X-x_i)^2+(Y-y_i)^2+H^2=v_i^2(t_i-t_0)^2, \tag{2}$$

where $i = 1, 2, \ldots, n$ are seismic station numbers; $x_i$ and $y_i$ are coordinates of the seismic stations; $t_0$ is the time at the focus, $t_i$ is the wave arrival time at station $i$, and $v_i$ is the effective wave velocity along the straight line connecting the hypocenter and the station $i$. Generally, $v_i$ are functions of the variables $X$, $Y$, $H$, $x_i$, $y_i$, but in problems involved in the choice of the optimal placement of seismic stations, the medium may be assumed, in first approximation, to be uniform, i.e., we can set $v_i = v = \text{const}$.

Depending on the problem statement, the unknown parameters can be the following: (A), $X$, $Y$, $H$; (B) $X$, $Y$, $H$, $v$; (C) $X$, $Y$, $H$, $t_0$; (D) $X$, $Y$, $H$, $v$, $t_0$. We will consider these four cases separately and show that they all can be reduced to a solution of linear system (1).

A. We introduce a new variable, $\xi=X^2+Y^2+H^2$ and transpose the terms with unknowns into the right-hand side, obtaining

$$Xx_i+Yy_i-\frac{1}{2}\xi=w_i, \tag{3}$$

where

$$i=1,2,\ldots,n\geq 3;\quad w_i=-\frac{1}{2}\left[v_i^2(t_i-t_0)^2-(x_i^2+y_i^2)\right].$$

B. We denote $V = v^2$. Then the parameters $X$, $Y$, $\xi$, $V$ can be defined from the linear equations system

$$Xx_i+Yy_i+\frac{1}{2}V(t_i-t_0)^2-\frac{1}{2}\xi=w_i, \tag{4}$$

where

$$i=1,2,\ldots,n\geq 4;\quad w_i=\frac{1}{2}(x_i^2+y_i^2).$$

C. We introduce the variable $\eta=X^2+Y^2+H^2-v^2t_0^2$. For $X$, $Y$, $\eta$ and $t_0$, we obtain the equations system

$$Xx_i+Yy_i-\frac{1}{2}\eta-t_0t_iv^2=w_i, \tag{5}$$

where

$$i=1,2,\ldots,n\geq 4;\quad w_i=-\frac{1}{2}(x_i^2+y_i^2-v^2t_i^2).$$

D. In this case, we introduce the variable $T = Vt_0$, and for $X$, $Y$, $V$, $\eta$, $T$, we have the following equations system:

$$Xx_i+Yy_i+\frac{1}{2}Vt_i^2-\frac{1}{2}\eta-Tt_i=w_i, \tag{6}$$

where

$$i=1,2,\ldots,n\geq 5;\quad w_i=\frac{1}{2}(x_i^2+y_i^2).$$

## 2. Majorant Estimates of Errors in the Determination of Near Earthquake Focal Coordinates

Equations linking the unknown parameters with the coordinates of seismic stations and wave arrival times can be written as the linear equations system (1). The estimates of the parameters, as has been mentioned, are written in the form

$$p=K^+u \tag{7}$$

where $K^+=(K^*K)^{-1}K^*$.

We will estimate the errors in the determination of the components of the vector $p$, assuming that the vector of free terms $w$ and the matrix are given with nonzero errors. Then, for the error of the vector $p$, we have the equation

$$K\Delta p = \Delta w - \Delta K p;$$

its solution is the vector

$$\Delta p = K^{+}(\Delta w - \Delta K p).$$

The solutions for the components of the vector $\Delta p$ are written as

$$\Delta p_j = k_j^{(+)}(\Delta w - \Delta K p),$$

where $k_j^{(+)}$ is a row vector of the matrix $K^{+}$. For the absolute value of the $j$-th component of the vector $\Delta p$, the inequality

$$|\Delta p_j| = |k_j^{(+)}(\Delta w - \Delta K p)| \leqslant \|k_j^{(+)}\| \cdot \|\Delta w - \Delta K p\|, \qquad (8)$$

will take place, which follows from the Cauchy-Buniakowski inequality, and $\|\cdot\|$ is the Euclidean norm.

We will now consider linear equations systems (3) - (6). We will assume that the errors in the elements of the matrix $K$, as well as in the right-hand sides of the equations, are caused solely by the errors in the wave arrival times $t_i$, and that the absolute values of these errors are $|\delta t_i| = \varphi_i |\Delta t|$. The weight factor $\varphi_i$ characterizes both the measurement quality at the station $i$ and the systematic error due to the deviation of the mean velocity $v$ from the effective velocity $v_i$ on the path between the hypocenter and the station $i$. As in the preceding section, we consider the four possible combinations of the sought-for parameters.

A. In this case, $\Delta K = 0$, and $\Delta w_i = v_i R_i \delta t_i$, where $R_i = v_i(t_i - t_0)$ is the hypocentral distance to the station $i$. We can readily see that

$$\|\Delta w\| \leqslant \left\{\sum_{i=1}^{n} |R_i v_i \varphi_i|^2\right\}^{1/2} |\Delta t| = \|Rv\varphi\| \cdot |\Delta t|$$

and therefore the estimates for $|\Delta p_j|$ appear as

$$|\Delta p_j| \leqslant \|k_j^{(+)}\| \cdot \|Rv\varphi\| \cdot |\Delta t|. \qquad (9)$$

B. For system (4), $\Delta w = 0$, and $(\Delta K p)_i = v^2(t_i - t_0)\delta t_i = vR_i\delta t_i$, so that for $|\Delta p_j|$ we again have estimate (9).

C. For system (5), $\Delta w_i = -v^2 t_i \delta t_i$, $(\Delta K p)_i = v^2 t_0 \delta t_i$ and $(\Delta w - \Delta K p)_i = -vR_i\delta t_i$. Therefore, in this case also, estimate (9) will take place.

D. For system (6), $\Delta w_i = 0$, and $(\Delta K p)_i = v^2 t_i \delta t_i - v^2 t_0 \delta t_i = vR_i \delta t_i$, so that estimate (9) is valid in this case as well.

We see that for all the alternatives considered, the Euclidean norm of the parameter error vector satisfies the inequality

$$\|\Delta p\| \leqslant \|K^{+}\| \cdot \|Rv\varphi\| \cdot |\Delta t|. \qquad (10)$$

Note that these estimates are unimprovable. This means that, for any fixed parameter values, one can always select the signs of the arrival time errors at the stations such as to obtain equality in (10). The right-hand side of inequality (10) therefore represents the maximum norm of the error. As has been mentioned in the Introduction, minimizing the maximum norm of the error is equivalent to $A$-, $E$-, and $D$-optimality tests, so that the problem of optimal placement of seismic stations can be posed as the problem of minimizing the objective function $J = \|K^{+}\| \cdot \|Rv\varphi\|$.

### 3. The Objective Function and Its Properties in Optimal Observation Systems

For the study of the objective function $J$, it is useful to examine the extremum properties of the cofactors $J_1 = \|K^{+}\|$ and $J_2 = \|Rv\varphi\|$.

1. $J_1 = \|K^{+}\|$. In [10], it was demonstrated that if $K$ is a nonsingular matrix, then for $\|K^{+}\|$ the following two-sided estimate takes place:

$$\left(\sum_{j=1}^{m} \frac{1}{\|k_j\|^2}\right)^{1/2} \leqslant \|K^{+}\| \leqslant \left(\frac{\prod_{j=1}^{m} \|k_j\|^2}{\det(K^{\mathrm{T}}K)} \sum_{j=1}^{m} \frac{1}{\|k_j\|^2}\right)^{1/2}. \qquad (11)$$

If the column vectors of the matrix $K$ are mutually orthogonal, then

$$\det(K^{\mathrm{T}}K) = \prod_{j=1}^{m} \|k_j\|^2$$

*and (11) degenerates into an equality:*

$$\|K^{+}\| = \left(\sum_{j=1}^{m} \frac{1}{\|k_j\|^2}\right)^{1/2}.$$

Minimizing $\|K^{+}\|$ is a difficult problem, so it is desirable to obtain some criteria of minimality of $\|K^{+}\|$ linked with the extremum properties of some other quantities.

*Consider the set of real matrices $K$ of the size $n \times m$* satisfying the condition $\|K\|^2 \leqslant C$. In [10], *the following theorem has been proved:* the determinant of the matrix $K^{\mathrm{T}}K$ reaches its maximum value at $\|K\|^2 = C$ *on matrices* with mutually orthogonal columns, and the same norm.

*From this theorem and estimate* (11) it follows that the minimum of $\|K^{+}\|$ and the maximum of *$\det(K^{\mathrm{T}}K)$ are achieved simultaneously.*

The constraint $\|K\|^2 \leqslant C$ is associated with the *constraints imposed on the region where the coordinates of the observation point are chosen (the planning region).* The planning region, in some cases, may correspond to a narrower set of matrices $K$, *for example* one defined by the relations $\|k_j\|^2 \leqslant c_j$, where $\sum_{j=1}^{m} c_j = C$. In that case, the following theorem takes place [10]: the determinant of the matrix $K^TK$ achieves its maximum at $\|k_j\|^2=c_j$ on matrices with mutually orthogonal columns.

These theorems thus define the properties for the matrix $K$ that are necessary for the observation system to be optimal in terms of the above tests. These results will be used in the subsequent analysis for determining the optimal systems of registration of near earthquakes with various initial data.

2. $J_2=\|Rv\varphi\|$. We will consider only Euclidean metric. In this case, $J_2$ is equal to

$$\left\{\sum_{i=1}^{n}[(X-x_i)^2+(Y-y_i)^2+H^2]v^2\varphi_i^2\right\}^{1/2}.$$

The minimum of $J_2$ as a function of $H$ is achieved at $H=0$, so that registration of surface sources is optimal for an area observation system.

Let us find the minimum of $J_2$ as a function of $X$ and $Y$. The point $M(X_0, Y_0, H=0)$, corresponding to the minimum of $J_2$ will be called optimal hypocenter. Differentiating $J_2$ with respect to $X$ and $Y$ and setting the derivatives equal to zero, we obtain

$$X_0=\sum_{i=1}^{n}\varphi_i^2x_i\Big/\sum_{i=1}^{n}\varphi_i^2,\quad Y_0=\sum_{i=1}^{n}\varphi_i^2y_i\Big/\sum_{i=1}^{n}\varphi_i^2.$$

The position of the optimal hypocenter thus coincides with the center of gravity of the system of points with the coordinates $x_i$, $y_i$ and the weights $\varphi_i^2$.

## 4. Statement of the Problem of the Optimum Geometry of a System of Seismic Stations

Suppose that, in the Earth's interior, a hypocentral region $\theta$ is given in which the hypocenters of earthquakes are distributed according to a certain pattern $\rho(X, Y, H)$; on the Earth's surface, a region $\Omega$ is defined in which the seismic stations for the registration of earthquakes from the region $\theta$ must be located. We will refer to the region $\Omega$ as the seismic experiment planning region.

The general problem of seismic experiment planning involves placing, in the region $\Omega$, seismic stations, in such a way that the function

$$\Phi=T+\alpha S[J(X, Y, H, x_i, y_i)],$$

which is called the loss function, be minimal. Here, the functional $S[J(X, Y, H, x_i, y_i)]$ defines the accuracy of determination of the hypocenter parameters, $\alpha$ is a normalizing factor, and $T$ describes the total cost of the experiment, which may be expressed, for example, in monetary terms. The cost $T$ is assumed to be proportional to the cost of measurements at all observation points [9]

$$T=\sum_{i=1}^{n}c_i\tau_i,$$

where $c_i$ is the cost of experiment $\tau_i$ conducted at the point $(x_i, y_i)$.

We will consider now the problem of seismic experiment planning for a given number of observation points, assuming that all $c_i=0$ and the functional $S$ has the form

$$S=\int_\theta P(X, Y, H, x_i, y_i)\rho(X, Y, H)d\theta,$$

where

$$\int_\theta \rho(X, Y, H)d\theta=1 \text{ and } \rho(X, Y, H)$$

and $\rho(X, Y, H)$ is a continuous function of the coordinates $X$, $Y$, $H$.

We will assume that the planning region $\Omega$ is infinite and all the measurements are of an equal accuracy, i.e., $\varphi_i=1$.

## 5. Optimal Geometry of Observation Points When Determining the Coordinates of Earthquake Hypocenters

Consider again the linear equations system (3). In accordance with the results of section 3 above, we will scale the coefficients of the matrix $K$ of the system so as to make the matrix of system (3) more balanced. We write the elements of the third column in the form $k_{i3}=-d/2$, where $d>0$ is the scale factor which has the dimension of the length. The linear equations system (3) then is rewritten as

$$Xx_i+Yy_i-\frac{d}{2}\xi_1=w_i, \tag{12}$$

where

$$\xi_1=\frac{\xi}{d}.$$

Generally, at $n\geq3$, we take for the solution of system (12) the solution of the normal equations system

$$K^tKp=K^tw. \tag{13}$$

A system of linear equations with a square matrix is known to have a unique solution if its determinant is nonzero. For system (13), the determinant is equal to zero in the case when all the observation points lie on a straight line. Obviously, this is the worst possible arrangement.

We will seek the solution of the problem in the form

$x_i = r_i \cos\psi_i$ and $y_i = r_i \sin\psi_i$, $i = 1, 2, \ldots, n \geq 3$.

For the columns of the matrix $K$ to be mutually orthogonal, we set

$$r_i = r, \quad \psi_i = \frac{2\pi}{n}(i-1).$$

Then the Euclidean norm of the generalized inverse matrix $K^+$ of system (12) will be equal to

$$\|K^+\| = 2\left[\frac{1}{n}\left(\frac{1}{r^2}+\frac{1}{d^2}\right)\right]^{1/2},$$

and the function $J(X, Y, H, x_i, y_i)$ is equal to:

$$J = 2v\left\{\left(\frac{1}{r^2}+\frac{1}{d^2}\right)(R_0^2+r^2)\right\}^{1/2},$$

where $R_0^2 = X^2+Y^2+H^2$.

The functional

$$S = \int_\theta J^2(X, Y, H, x_i, y_i)\rho(X, Y, H)\,d\theta = 2v^2\left[\frac{1}{r^2}+\frac{1}{d^2}\right][\mathscr{E}(R_0^2)+r^2]$$

will be smaller, the smaller the mean value of $R_0^2$. Obviously, $S$ is minimal also when the center of the observation system coincides with the center of distribution of the earthquake epicenters.

We will find the minimum of $S$ as a function of $r$. Differentiating $S$ with respect to $r$ and setting the derivative equal to zero, we will eventually obtain

$$r = \sqrt{d\sqrt{\mathscr{E}(R_0^2)}}. \tag{14}$$

In order for the matrix $K$ of system (12) to be balanced in columns, the scale factor $d$ should be taken as

$$d = \sqrt{2}r.$$

Then, substituting this equality into (14), we find

$$r = \sqrt{2\mathscr{E}(R_0^2)}. \tag{14'}$$

The estimate of the error in the evaluation of the vector $p$ will appear as

$$\|\Delta p\| \leq v\left\{3\left[\frac{R_0^2}{\mathscr{E}(R_0^2)}+2\right]\right\}^{1/2}|\Delta t|. \tag{15}$$

Obviously, this distribution of $n$ seismic stations is optimal.

As an example, we will consider the case when the region $\theta$ is a circular cylinder (with the element perpendicular to the day surface) of the height $H_c$ and the base radius $r_c$. Earthquake hypocenters are distributed uniformly throughout the volume of this cylinder. The mean $R_0^2$ in this case is equal to

$$\mathscr{E}(R_0^2) = \frac{r_c^2}{2}+\frac{H_c^2}{3}$$

and therefore

$$r = \sqrt{r_c^2+\frac{2}{3}H_c^2}.$$

If $H_c = 0$, that is, all the earthquakes occur on the surface, $r = r_c$.

From the analysis of estimate (15) it follows that the maximum error in the evaluation of earthquake hypocenter coordinates for an optimal observation system is independent of the number of recording stations. If the cost of each observation point is taken into account, a system consisting of a minimal number of points, i.e., three, will be optimal.

Suppose that an observation system consists of just three stations. Differentiating the functional $S$ with respect to the variables $x_i$ and $y_i$ ($i = 1, 2, 3$) and setting the derivatives equal to zero, we can readily demonstrate that the resulting optimal placement of the seismic stations, accurate to rotation of the observation system around the center of the circle by an arbitrary angle $\psi_0$, is unique.

In the general case, at $n > 3$, this solution is not the only one.

As an illustration, consider an observation system comprised of more than three points, with $n-1$ points located on a circle of the radius $r$, and the point $n$ placed at the center of the circle, i.e., consider a system of points with the coordinates

$$x_i = r\cos\frac{2\pi}{n-1}(i-1), \quad y_i = r\sin\frac{2\pi}{n-1}(i-1),$$

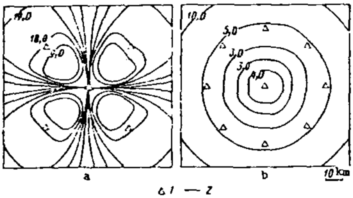
$$x_n = y_n = 0, \quad i = 1, 2, \ldots, n-1 \geq 3.$$

With this arrangement of the observation points, the column vectors of the matrix $K$ of system (12) will also be mutually orthogonal, and the estimate of the error of the vector $p$ will appear as

$$\|\Delta p\| \leq 2v\left\{\left[\frac{n}{(n-1)r^2}+\frac{1}{d^2}\right]\left[R_0^2+\frac{n-1}{nr^2}\right]\right\}^{1/2}|\Delta t|. \tag{16}$$

The minimal value of the estimate will take place at

$$r^2 = \frac{n}{n-1}dR_0 = 2\frac{n}{n-1}R_0^2$$




Δ 1 — 2

Fig. 1. Distribution of errors in the estimation of epicentral distances for two observation systems: 1) seismic stations; 2) error isolines, km.

and for

$$r^2=2\frac{n}{n-1}\mathscr{E}(R_0^2)$$

it will appear as

$$\|\Delta p\|\leqslant v\left\{3\left[\frac{R_0^2}{\mathscr{E}(R_0^2)}+2\right]\right\}^{1/2}|\Delta t|. \qquad (16')$$

For the matrix of system (12) to be balanced, we set

$$d^2=2\frac{n-1}{n}r^2.$$

We see that estimates (15) and (16') coincide and that the resulting distribution of the observation points is also optimal.

## 6. Optimal Geometry of Observation Points When Determining the Coordinates of Earthquake Hypocenters, the Propagation Velocity of Seismic Waves, and the Time at the Focus

Proceeding from the above method of study of Eqs. (4) - (6), one can readily determine optimal and nonoptimal observation systems in cases when either the seismic wave propagation velocity or the origination time of the earthquake (the time at the focus), or both, are unknown, as are the coordinates of the hypocenters.

When the seismic wave velocity or the focal time are unknown, the worst placement of the seismic stations is one where all the observation points lie on a straight line or a circle.

An observation system where $n - 1$ points are in a circle and the point $n$ is at the center of the circle is optimal.

If we want to determine the coordinates of the hypocenter, the seismic wave propagation velocity, and the focal time simultaneously, the worst placement of the observation points is one where all the recording stations are on a straight line or a circle. Other nonoptimal observation systems apparently exist for this case as well.

Figures 1a and b give the isolines of the maximum errors of epicentral distances for two observation systems with the error of $t_i$ equal to $|\Delta t|=0.05$ s. Figure 1a, illustrating the case of a system of five stations, clearly shows a trend of the observation system. The minimum number of errors occurs on the directions defined by the straight lines drawn through three stations. The maximum number of errors occurs on the directions defined by the lines drawn through the central station at an angle of 45° to the minimum error directions. Considering the determinant det $(K^TK)$, where $K$ is the matrix of linear equations system (6), it can readily be demonstrated that for $n$ = 5 the equations system is singular on these directions.

## CONCLUSIONS

The nonstatistical method of analysis of experiment planning problem suggested in the paper yields a solution of the problem of optimal placement of seismic stations when determining the coordinates of the hypocenters from various initial data. In some of the cases the solution can be produced in an explicit form. An important feature of seismic observations is the fact that a system of stations optimal for certain initial data ceases to be optimal if the data are different. The larger the number of unknown parameters included in the initial equations system, the "worse" the observation system becomes. It has been calculated in particular that for an optimal observation system, the maximum of errors in the determination of earthquake hypocenter coordinates when the focal time and the seismic wave velocity are known are smaller by several times than the maximum number of errors when the seismic wave velocity is known but the focal time is unknown, all other conditions being equal. It should also be noted that identifying nonoptimal observation systems is also of a major importance for practical applications.

The problem of an optimal placement of a network of seismic stations examined in the paper refers to an infinite region. Where the planning region contains an epicentral region and is relatively large, these results can be used as a point of departure in planning seismic observations.

Other problem statements are also possible. For example, in the region $\Omega$ $n$ observation points have already been placed. The objective is to proceed from the given distribution $\rho(X, Y, H)$ and find the optimal placement of additional $m$ points in $\Omega$ (and probably the number of such points) according to how they meet the tests described in the paper. This problem is solved by integer methods minimizing a functional of the objective function.

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