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Distributions of Density and Elastic Parameters in the Earth

V. Yu. Burmin

Schmidt Institute of Physics of the Earth, Russian Academy of Sciences, Bol'shaya Gruzinskaya ul. 10, Moscow, 123995 Russia

e-mail: burmin@ifz.ru

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Abstract—The velocity curve previously obtained for P waves in the Earth is used to determine the distributions of the density and elastic parameters. The density distribution in the new model differs from that in the standard PREM model only in the inner core. The distributions of the bulk and shear moduli can differ, depending on physical processes in the Earth. In particular, the bulk modulus can have a negative jump at the outer–inner core boundary, whereas the shear modulus can differ from zero in the lower part of the outer core.

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INTRODUCTION

The determination of the distribution of the density and elastic parameters in the Earth is one of the major problems in geophysics. This problem has been extensively studied, but an unambiguous solution has not been obtained so far. This is due to two main causes. The first is the nonuniqueness of the problem solution in the presence of density discontinuities within the Earth. The second cause is the imperfection of velocity models of the Earth. At present, researchers continue to refine the distribution of seismic velocities even in a spherically symmetrical Earth. Below, we consider the problem without reviewing numerous publications on this subject. The history of relevant investigations and a comprehensive bibliography can be found in [Bullen, 1963, 1975; Gutenberg, 1959; Zharkov and Trubitsyn, 1980; Zharkov, 1983; Kalinin and Sergeeva, 1971, 1975; Magnitsky, 1965; Pankov and Zharkov, 1967].

The present work was primarily initiated by the refined longitudinal velocity distribution in the Earth published in [Burmin, 2004a, 2004b]. The updatings are most significant in the middle mantle, the base of the outer core, and the F zone of the core [Burmin, 2004a]. Figures 4 and 5 show the refined velocity curve. Its distinctive features are the presence of waveguides in the upper and middle mantle, an increased velocity gradient in the lowermost outer core, and a waveguide in the F zone of the core. This behavior of the velocity curve requires appropriate refinement and interpretation of the behavior of the density and elastic parameters in the Earth.

BASIC PRINCIPLES

The basic equation for determining the density $\rho(z)$ within the Earth is the well-known Williamson–Adams equation [Williamson and Adams, 1923], which can be written as

$$d\rho = \frac{g(z)\rho(z)}{\Phi(z)}dz. \qquad (1)$$

Here, $g(z) = G\frac{M - m(z)}{(R - z)^2}$ is the gravitational acceleration at the depth z; G is the gravitational constant; M is the mass of the Earth; $m(z) = 4\pi\int_0^z \rho(\xi)(R - \xi)^2 d\xi$ is the mass of a spherical layer of the thickness $z = R - r$; R is the Earth's radius; $r = R - z$ is the distance between the Earth's center and the point under consideration; $\Phi(z)$ is the so-called seismic parameter, equal to $\Phi(z) = v_P^2 - \frac{4}{3}v_S^2 = \frac{k_S}{\rho}$; $v_P = \sqrt{\frac{k_S + \frac{4}{3}\mu}{\rho}}$ and $v_S = \sqrt{\frac{\mu}{\rho}}$ are the P and S wave velocities, respectively; k_S is the adiabatic bulk modulus; and μ is the shear modulus.

Equation (1) was obtained for a spherically symmetrical Earth in hydrostatic equilibrium. Moreover, Eq. (1) is valid for a continuous density distribution with depth. In the case of discontinuities of the function $\rho(z)$, appropriate density values should be specified at the discontinuous boundaries. Furthermore, Eq. (1) is valid for a homogeneous medium, i.e., a medium in which the chemical composition of rocks does not vary with depth. In practice, however, it turns out to be valid for a heterogeneous medium with a continuously varying composition.

Now, we describe other basic equations. Hooke's law for the Earth's interior has the form

$$dp = -k_T\frac{dV}{V} = k_T\frac{d\rho}{\rho}, \qquad (2)$$

where k_T is the isothermal bulk modulus and V is the unit volume. Note that the total increment $d\rho$ is present in (2).

Formula (2) relates the density change at the point M of a unit volume $d\Omega$ to the pressure change within this volume and is valid for both homogeneous and heterogeneous media, as follows from the local nature of this relation. Since the density ρ depends on both the depth-dependent pressure and the chemical composition, it is obvious that the bulk modulus k_T is likewise a function of the pressure and chemical composition.

Because the Earth is assumed to be in hydrostatic equilibrium, the following hydrostatic equation is valid:

$$dp = g(z)\rho(z)dz. \qquad (3)$$

Equation (3) describes the dependence of the pressure change at the point M of the same volume $d\Omega$ on the change in the mass of this volume and is also valid for chemically heterogeneous media.

We have from (2) and (3):

$$\frac{k_T}{\rho}d\rho = g(z)\rho(z)dz.$$

Expressing k_T through the k_S value, which is determined from P and S velocities, we obtain the relations for the adiabatic bulk modulus k_S

$$k_S = \rho(z)\Phi(z) \qquad (4)$$

and

$$\frac{1}{k_T} = \frac{1}{k_S}\frac{C_p}{C_V} = \frac{1}{k_S} + \frac{TV}{C_p}\beta^2 = \frac{1+\beta T\Gamma}{k_S}, \qquad (5)$$

where C_p is the specific heat at a constant pressure; C_V is the specific heat at a constant volume; $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$ is the coefficient of volume expansion; $V = \frac{1}{\rho}$; T is the temperature in absolute degrees; and $\Gamma = \frac{\beta V k_S}{C_V}$ is the Grüneisen parameter, which is of the order of 2 for the majority of materials [Svenson, 1963].

Substituting (4) and (5) into (3), we obtain

$$d\rho = \eta\frac{g(z)\rho(z)}{\Phi(z)}dz, \qquad (6)$$

where $\eta = \frac{k_S}{k_T} = \frac{C_p}{C_V} = 1 + \frac{k_S T}{C_p\rho}\beta^2 = 1 + \beta T\Gamma \geq 1$.

Thus, Eq. (1) in the general case should be replaced by Eq. (6), which includes the relation $\frac{k_S}{k_T} = \frac{C_p}{C_V}$. However, the quantities C_p and C_V in solid bodies differ insignificantly and it is possible to use only Eq. (1) to determine the density distribution, particularly because the real distribution of C_p and C_V within the Earth is unknown.

In addition to the fact that, when deriving formula (1), the value k_T is replaced by k_S, there is yet another problem, which was noted by Magnitsky [1952]. This problem is related to the variation in the chemical composition in the Earth's interior. The point is that the value k_S in formula (1) does not take into account the variation in the chemical composition, whereas values derived from observations are affected by these variations.

Kalinin [1972] derived an equation determining the density variation with depth in a heterogeneous Earth taking into account the variation in the chemical composition. According to the results of his work, Eq. (1) should have the form

$$\frac{d\rho}{dz} = \frac{g\rho}{\Phi} + \beta\rho t = \rho\left(\frac{g}{\Phi} + \beta t\right), \qquad (7)$$

where $t = \left(\frac{dT}{dz}\right)_K - \frac{dT}{dz} \geq 0$ is the preconvective temperature gradient.

In the particular case of $t = 0$, Eq. (7) coincides with Eq. (1). This corresponds to the case when the chemical composition of the Earth's interior does not vary with depth [Kalinin, 1972].

Kalinin [1972] drew the conclusion that the influence of heterogeneities, or, more specifically, a variation in the chemical composition, on the density distribution in the Earth affects, first, the quantity Φ, which is determined from seismic observations of the Earth's real material, and, second, the quantity t, which in reality is fairly small. Below, we present estimates of βt in all depth ranges in the Earth and compare them with the values of $\frac{g}{\Phi}$ at the same depths.

Bullen [1963, 1975] also examined the influence of the heterogeneity of the chemical composition in the Earth on the density distribution. In these and his other works, he presents an equation similar to Eq. (6), but the value η in this case is $\eta = \frac{dk_S}{dp} - g^{-1}\frac{d\Phi}{dz}$ and "is an indicator of the deviation of the medium from chemical homogeneity." It is not difficult to show that the values η in Eq. (6) and in Bullen's works are the same.

Indeed, we have for k_S:

$$k_S = \Phi\rho \quad \text{and} \quad \frac{dk_S}{dz} = \frac{dk_S}{dp}\frac{dp}{dz}.$$

This yields $\frac{dk_S}{dp} = \frac{dk_S}{dz}\left(\frac{dp}{dz}\right)^{-1}$. Then, considering that $g = \frac{1}{\rho}\frac{dp}{dz}$, we obtain

$$\eta = \frac{dk_S}{dz}\left(\frac{dp}{dz}\right)^{-1} - \rho\left(\frac{dp}{dz}\right)^{-1}\frac{d}{dz}\left(\frac{k_S}{\rho}\right)$$
$$= \frac{dk_S}{dz}\left(\frac{dp}{dz}\right)^{-1} - \frac{dk_S}{dz}\left(\frac{dp}{dz}\right)^{-1} + \frac{k_S}{\rho}\frac{d\rho}{dz}\left(\frac{dp}{dz}\right)^{-1} = \frac{k_S}{k_T}.$$

This means that η is not an indicator of the chemical heterogeneity of the medium but depends on the coefficient of thermal expansion β and temperature T.

The solution of Eq. (1) must satisfy the relations determining the Earth's mass and moment of inertia about the rotation axis

$$M = 4\pi\int_0^R \rho(\xi)(R-\xi)^2 d\xi,$$
$$J = \frac{8}{3}\pi\int_0^R \rho(\xi)(R-\xi)^4 d\zeta, \qquad (8)$$

where the values of M and J are determined from observations.

After the distribution $\rho(z)$ is found, the corresponding values of the bulk modulus k_S, shear modulus μ, gravitational acceleration g, and pressure p are determined by the formulas

$$k_S(z) = \Phi(z)\rho(z), \quad \mu(z) = v_S^2(z)\rho(z),$$
$$g(z) = G\frac{M - m(z)}{(R-z)^2}, \quad p(z) = \int_0^z g(\xi)\rho(\xi)d\xi. \qquad (9)$$

Henceforward, the solution of Eq. (1) for determining the density distribution from observed P and S wave velocities is referred to as the inverse problem, in contrast to the mathematical modeling of the density distribution, which will be referred to as the forward problem.

NUMERICAL SOLUTION OF THE FORWARD PROBLEM FOR A HETEROGENEOUS GRAVITATING SPHERE

Numerical modeling of a heterogeneous medium consists in the determination of the variation in the density of material in the planet's interior or, more specifically, in a gravitating sphere, as a function of the depth or radius, taking into account the excess density due to the overburden pressure and the chemical heterogeneity of the medium. In this paper, three cases are examined. In the first case, no additional density is produced by variation in the rock composition, i.e., a chemically homogeneous medium is examined. In the second case, the additional density due to a depth increase is positive. In the third case, the additional density due to a depth increase is negative.

The density distribution in a gravitating sphere was also modeled by solving Eq. (1); however, unlike the inverse problem, in which the density is determined from the surface toward the center of the sphere, the density distribution in the forward problem is determined from the center toward the surface of the sphere.

In solving the problem, two variants are possible. In the first variant, the elastic modulus k remains constant and does not depend on pressure. In the second variant, k depends on pressure and therefore varies with depth.

In the first case, Eq. (1) is written as

$$\frac{d\rho}{\rho^2} = -d\left(\frac{1}{\rho}\right) = -\frac{g}{k}dr, \quad \text{where} \quad g = G\frac{m(r)}{r^2},$$

or, in the discrete form,

$$\frac{1}{\rho_i} - \frac{1}{\rho_{i-1}} = \frac{g_i}{k}(r_i - r_{i-1}),$$

where $i = 2, \ldots, n$; $m_1 = \frac{4}{3}\pi\rho_1 r_1^3$; ρ_1 is the density at the center of the Earth; and $g_i = G\frac{m_{i-1}}{r_i^2}$.

Hence,

$$\rho_i = \frac{\rho_{i-1}}{(1+\rho_{i-1}f_i)}, \qquad (10)$$

where $f_i = \frac{g}{k}(r_i - r_{i-1})$ and $m_i = m_{i-1} + 4\pi\rho_i r_i^2(r_i - r_{i-1})$.

In the second case, it is necessary to consider the pressure dependence of the elastic modulus k. To do this, one can use the known formula relating compressibility with pressure [Birch, 1952; Svenson, 1963]

$$\frac{\Delta V}{V_0} = \frac{V - V_0}{V_0} = -ap + bp^2, \qquad (11)$$

where $V = \frac{1}{\rho}$ is the specific volume and a and b are constants.

From (11), we have $\frac{dV}{V_0} = -(a - 2bp)dp$ and, considering that $k = \rho\frac{dp}{d\rho} = -V\frac{dp}{dV}$, we obtain

$$k = -V\frac{dp}{dV} = \frac{\rho_0}{\rho}\frac{1}{a-2bp}.$$

Thus, in the second case, Eq. (1) can be written as

$$\frac{d\rho}{\rho^3} = -\frac{1}{2}d\left(\frac{1}{\rho^2}\right) = -g\frac{(a-2bp)}{\rho_0}dr,$$

where ρ_0 is the density at $p = 0$.

The discrete variant is

$$\frac{1}{\rho_i^2} - \frac{1}{\rho_{i-1}^2} = -2g_i\frac{(a-2bp_i)}{\rho_0}(r_i - r_{i-1})$$

and $$\rho_i = \left\{\frac{1}{\rho_{i-1}^2} - 2g_i\frac{(a-2bp_i)}{\rho_0}(r_i - r_{i-1})\right\}^{-1/2}.$$

In order to determine the constants a and b, it is necessary to specify the values of the density, pressure, and shear modulus at the surface of the body and at its center. Then, at $z = 0$, we obtain $\rho = \rho_0, p = p_0 = 0, k = k_0$, and $a = 1/k_0$. At $z = R$, we have $\rho = \rho_m$, $p = p_m = 0$, $k = k_m$, and $b = \frac{1}{2p_m}\left(\frac{1}{k_0} - \frac{1}{k_m}\frac{\rho_0}{\rho_m}\right)$.

It is clear that the second case corresponds better to the real distribution of elastic parameters; for this reason, it is examined in this paper. First, the problem was solved for a homogeneous planet. The density, pressure, and bulk modulus at the center of the planet were set equal to $\rho_m = 12.8$ g/cm^3, $p_m = 3.512 \times 10^{11}$ Pa, and $k_m = 1.4 \times 10^{12}$ Pa (these values are assumed to be characteristic of the center of the Earth). The inner core is assumed to consist mainly of iron. At the surface of the planet ($\rho_0 = 0$ Pa), the iron values were taken for the density and the bulk modulus: $\rho_0 = 7.874$ g/cm^3 and $k_0 = 1.7 \times 10^{11}$ Pa. The calculated values are presented in Fig. 1, which also shows the inversion results for the density, pressure, and elastic moduli from the seismic wave velocities and the density at the surface of the planet. Both density curves virtually coincide and they are, therefore, indistinguishable in the figure.

Then, the density of a chemically heterogeneous medium with an excess density was determined. The additional density was specified as follows. A value of 0.005 g/cm^3 was added at 10-km intervals to each value of the density determined for a homogeneous medium. The resulting values were then used to solve the forward and inverse problems. The numerical results are presented in Fig. 2. Both density curves shown in the figure coincide, implying that Eq. (1) is valid for a heterogeneous medium.

The same procedure was applied to the medium with negative additional densities. The numerical results shown in Fig. 3 indicate that rocks composing deep layers under normal conditions can have lower densities compared to rocks of the overlying layers.

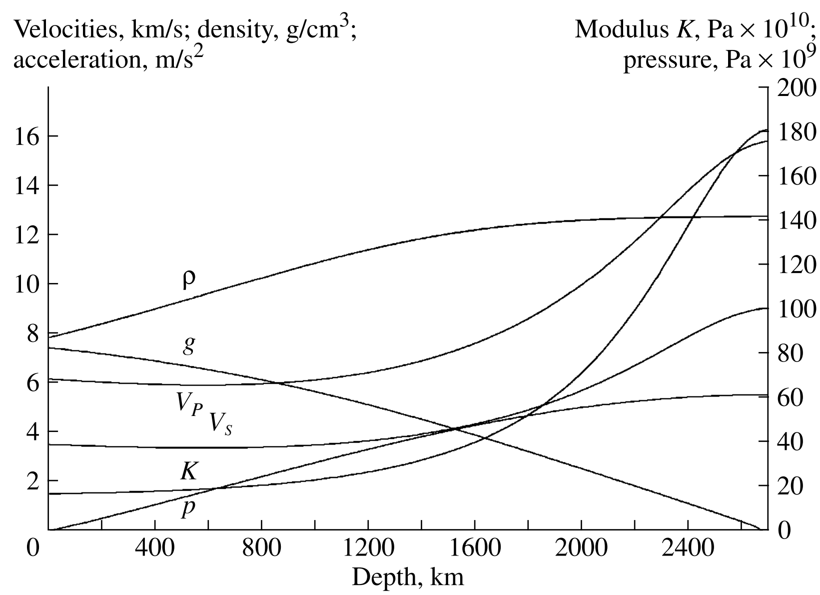

Fig. 1. Distributions of geophysical parameters in a homogeneous gravitating sphere.

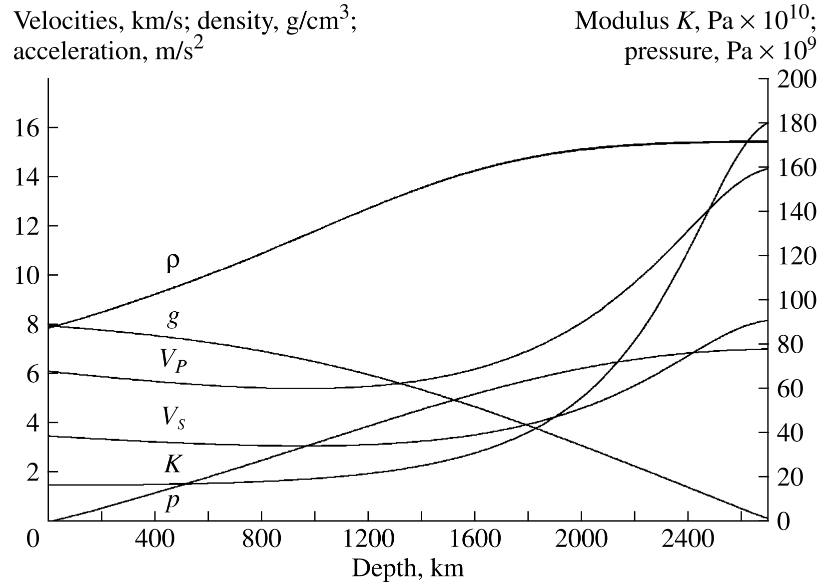

Fig. 2. Distributions of geophysical parameters in a gravitating sphere with an excess density.

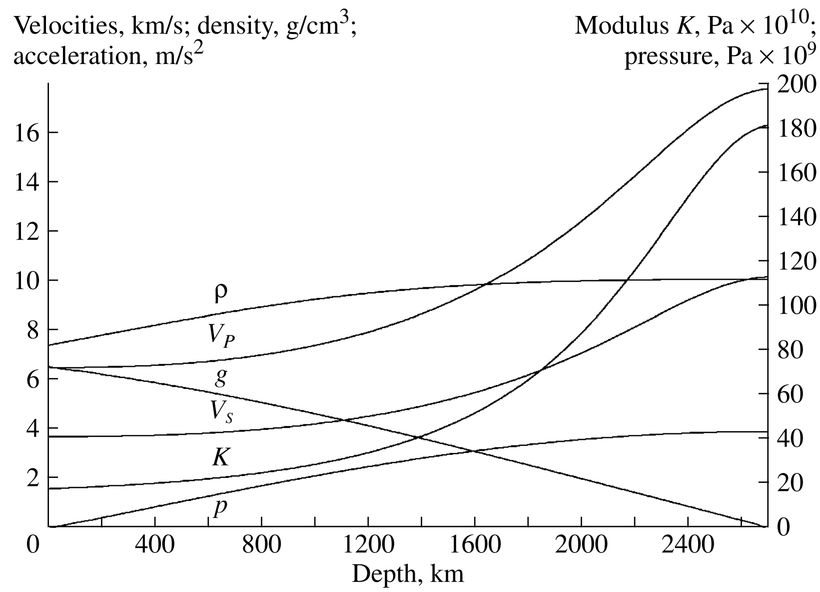

Fig. 3. Distributions of geophysical parameters in a gravitating sphere with a density deficit.

NUMERICAL SOLUTION OF THE INVERSE PROBLEM FOR THE REAL EARTH

Equations (1) with respect to the function $\rho(z)$ satisfying relations (8) are easily solved numerically. We follow the scheme described below. For the mantle and outer core, we solve Eq. (1) independently of relation (8) for J. For the inner core, we solve Eqs. (1) and (8) simultaneously but independently. The density in Eqs. (8) is approximated by a linear function; fitting the density values at the boundaries, we make the density curves in the inner core derived from Eqs. (1) and (8) coincide. The values of the mass and moment of inertia are specified in accordance with the available data: $M = 5.976 \times 10^{24}$ kg and $J = 8.023 \times 10^{37}$ kg m^2. The densities at the boundaries are initially taken from the PREM model [Dziewonski and Anderson, 1981].

To solve Eq. (1) numerically, we discretize it in accordance with the specified grid of depths z_i and v_{Pi}, v_{Si} ($i = 1, 2, \ldots, n$). In this case, (1) can be written as

$$\rho_i - \rho_{i-1} = \rho_{i-1}\frac{g_{i-1}}{\Phi_i}(z_i - z_{i-1}), \tag{12}$$

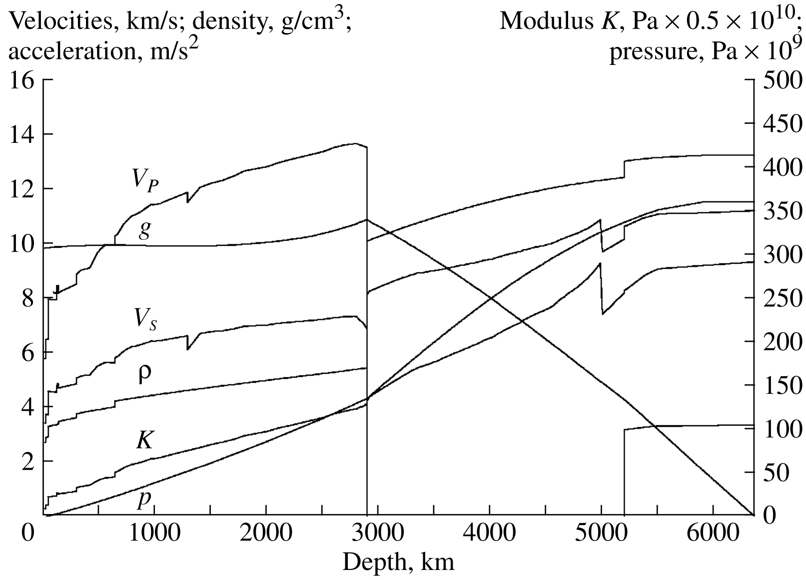

Fig. 4. Distributions of geophysical parameters in the Earth with a nonmonotonic variation in the bulk modulus in the core.

where $i = 2, 3, \ldots, n$; $z_1 = 0$; and ρ_1 and g_1 are values of the density of the medium and the gravitational acceleration at the Earth's surface that are specified a priori. Furthermore, for all density discontinuity depths z_i, it is necessary to specify the corresponding values of ρ_i.

From (12), we have

$$\rho_i = \rho_{i-1}\left[1 + \frac{g_i}{\Phi_i}(z_i - z_{i-1})\right], \tag{13}$$

where $g_i = G\dfrac{M - m_i}{(R - z_{i-1})^2}$.

The mass m_i of the layer from the Earth's surface to the depth z_i is determined by the formula from (8) under the assumption that the density in each layer $z_i - z_{i-1}$ varies linearly with the gradient $\gamma_i = \dfrac{\rho_i - \rho_{i-1}}{z_i - z_{i-1}}$:

$$m_i = m_{i-1} + \pi\left\{\frac{4}{3}[\rho_{i-1} + \gamma_i(R - z_{i-1})] \times [(R - z_{i-1})^3 - (R - z_i)^3] - \gamma_i[(R - z_{i-1})^4 - (R - z_i)^4]\right\}.$$

Since relation (13) at each step of calculations contains the unknown value m_i, the density is determined using the following iterative procedure. At the first step, we assume $\gamma_i = 0$ and determine m_i and ρ_i under this assumption. At subsequent steps of calculations, we assume $\gamma_i \geq 0$. This procedure generally converges after two steps.

The moment of inertia is determined under the same assumptions from the relation

$$J = \frac{4}{9}\pi\sum_{i=2}^{n}\left\{\frac{6}{5}[\rho_i + \gamma_i(R - z_{i-1})] \times [(R - z_{i-1})^5 - (R - z_i)^5] - \gamma_i[(R - z_{i-1})^6 - (R - z_i)^6]\right\}.$$

RESULTING DISTRIBUTIONS OF DENSITY AND OTHER PHYSICAL PARAMETERS IN THE EARTH

The results of determining the density, bulk modulus, pressure, and gravitational acceleration from *P* and *S* velocities in the Earth's interior are presented in Figs. 4 and 5. Figure 4 shows the curves of the geophysical parameters under the assumption that the *S* wave velocity in the Earth's outer core is equal to zero. In this case, as can be seen from the figure, the bulk modulus drops by a jump at the outer–inner core boundary. This behavior of the shear modulus can be due to at least two causes.

The first cause is the rearrangement of the two outer electron shells of iron atoms under pressure. A significant pressure can cause the transition of one or two electrons from the level 4*s* (shell *N*) to the level 3*d* (shell *M*), thereby increasing the distance between ions in the iron crystal. Zhdanov [1961, p. 206] gives the fol-

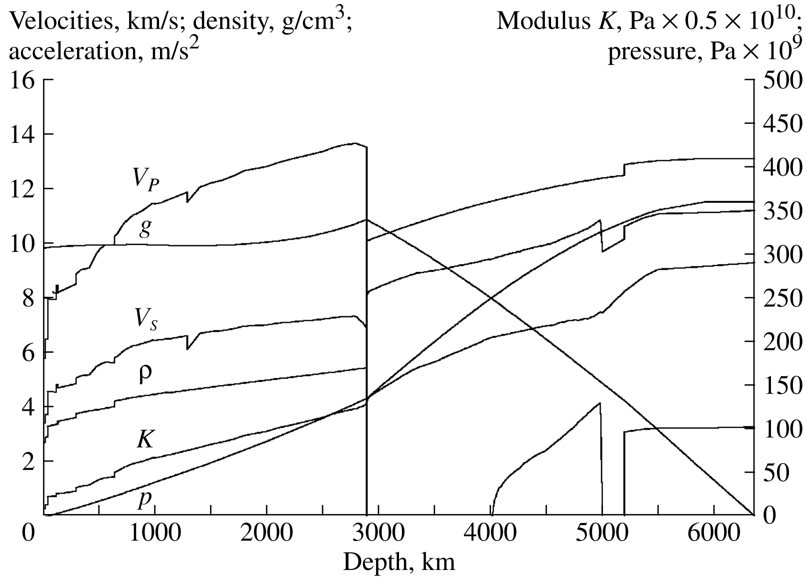

Fig. 5. Distributions of geophysical parameters in the Earth with a monotonic variation in the bulk modulus in the core.

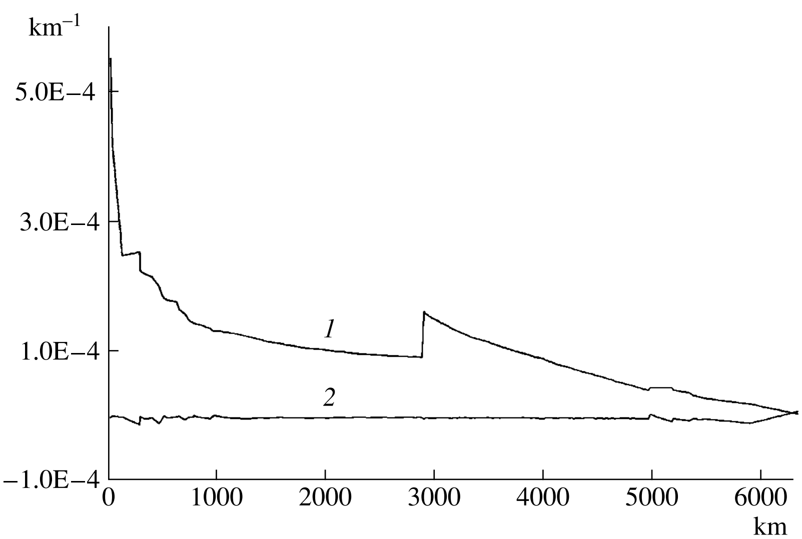

Fig. 6. Distribution of (*1*) $\frac{g}{\Phi}$ and (*2*) $\left(\frac{1}{\rho}\frac{d\rho}{dz} - \frac{g}{\Phi}\right)$.

lowing formula for calculating the compressibility β of a metal:

$$\beta = \frac{9vr}{2ANz^2e^2},$$

where r is the distance between ions in the crystal, v is the volume of a mole of the crystal, A is the Madelung constant, N is the Avogadro number, z is the valence of the metal, and e is the electron charge. This formula implies that, with increasing distance between ions in the crystal, the compressibility increases or, accordingly, the bulk modulus k decreases.

The second cause is an increase in the Ni concentration with depth in the Earth's core. A Fe–Ni alloy is known to have a lower bulk modulus compared to iron or nickel.

Figure 5 shows the curves of geophysical parameters obtained under the assumption that the bulk modulus is a monotonic function of depth. To meet this requirement, the shear modulus in the outer core must be nonzero beginning from a depth of about 400 km, Distribution of physical parameters in the Earth

z	V_P	V_S	ρ	g	P	K	μ
.0000	6.0100	3.4740	2.6000	9.8160	.0000E+00	.5207E+11	.3138E+11
5.2200	6.0100	3.4735	2.6067	9.8179	.1336E+09	.5222E+11	.3145E+11
41.1300	6.1000	3.5221	2.6530	9.8500	.1072E+10	.5484E+11	.3291E+11
41.1300	7.9600	4.5961	2.8000	9.8500	.1072E+10	.9855E+11	.5915E+11
116.8200	7.9300	4.5694	2.8595	9.9081	.3216E+10	.1002E+12	.5970E+11
116.8200	8.4500	4.8690	3.2000	9.9081	.3216E+10	.1273E+12	.7586E+11
132.4200	8.4600	4.8728	3.2124	9.9157	.3713E+10	.1282E+12	.7628E+11
132.4200	8.2000	4.7230	3.4000	9.9157	.3713E+10	.1275E+12	.7584E+11
294.5400	8.5000	4.8746	3.5449	9.9672	.9442E+10	.1438E+12	.8423E+11
294.5400	8.8700	5.0868	3.7000	9.9672	.9442E+10	.1635E+12	.9574E+11
301.7700	8.8900	5.0972	3.7060	9.9685	.9709E+10	.1645E+12	.9629E+11
344.9000	9.0300	5.1715	3.7418	9.9761	.1132E+11	.1717E+12	.1001E+12
412.5900	9.1000	5.2022	3.7965	9.9877	.1389E+11	.1774E+12	.1027E+12
476.1100	9.6400	5.5016	3.8477	9.9983	.1633E+11	.2023E+12	.1165E+12
508.0100	9.7900	5.5825	3.8709	10.0037	.1756E+11	.2102E+12	.1206E+12
519.6600	9.8300	5.6035	3.8792	10.0056	.1802E+11	.2124E+12	.1218E+12
555.0400	9.9500	5.6666	3.9042	10.0116	.1940E+11	.2194E+12	.1254E+12
640.0000	9.9500	5.6538	3.9625	10.0263	.2277E+11	.2234E+12	.1267E+12
640.0000	10.2700	5.8357	4.3652	10.0263	.2277E+11	.2622E+12	.1487E+12
663.1700	10.3400	5.8718	4.3820	10.0225	.2379E+11	.2671E+12	.1511E+12
717.6500	10.7000	6.0675	4.4208	10.0136	.2620E+11	.2891E+12	.1628E+12
742.4100	10.8200	6.1315	4.4375	10.0096	.2730E+11	.2971E+12	.1668E+12
765.6100	10.9200	6.1844	4.4528	10.0058	.2834E+11	.3039E+12	.1703E+12
794.1202	11.0100	6.2307	4.4713	10.0011	.2961E+11	.3106E+12	.1736E+12
797.4800	11.0100	6.2301	4.4735	10.0006	.2976E+11	.3108E+12	.1736E+12
805.0300	11.0200	6.2346	4.4783	9.9994	.3010E+11	.3118E+12	.1741E+12
810.3702	11.0300	6.2393	4.4817	9.9985	.3034E+11	.3126E+12	.1745E+12
877.6400	11.1800	6.3130	4.5243	9.9878	.3338E+11	.3251E+12	.1803E+12
954.8300	11.4000	6.4241	4.5719	9.9759	.3690E+11	.3426E+12	.1887E+12
970.0300	11.4300	6.4385	4.5811	9.9737	.3759E+11	.3453E+12	.1899E+12
973.8102	11.4300	6.4378	4.5834	9.9731	.3777E+11	.3455E+12	.1900E+12
983.9100	11.4200	6.4305	4.5895	9.9716	.3823E+11	.3455E+12	.1898E+12
1035.3600	11.4500	6.4387	4.6204	9.9643	.4060E+11	.3503E+12	.1915E+12
1148.9400	11.6400	6.5261	4.6873	9.9496	.4590E+11	.3689E+12	.1996E+12
1287.9000	11.8600	6.6253	4.7665	9.9352	.5248E+11	.3915E+12	.2092E+12
1287.9200	11.5000	6.1242	4.7665	9.9352	.5248E+11	.3920E+12	.1788E+12
1400.0000	12.0500	6.7118	4.8290	9.9272	.5785E+11	.4111E+12	.2175E+12
1500.0000	12.2000	6.7778	4.8837	9.9234	.6270E+11	.4278E+12	.2243E+12
1600.0000	12.3000	6.8157	4.9373	9.9233	.6760E+11	.4412E+12	.2294E+12
1700.0000	12.4500	6.8810	4.9905	9.9276	.7255E+11	.4585E+12	.2363E+12
1800.0000	12.6500	6.9735	5.0427	9.9369	.7756E+11	.4800E+12	.2452E+12
2000.0000	12.8000	7.0201	5.1412	9.9736	.8782E+11	.5045E+12	.2534E+12
2100.0000	12.9500	7.0842	5.1918	10.0028	.9301E+11	.5233E+12	.2606E+12
2200.0000	13.0500	7.1208	5.2416	10.0402	.9827E+11	.5383E+12	.2658E+12
2300.0000	13.1500	7.1571	5.2911	10.0871	.1036E+12	.5536E+12	.2710E+12

Table. (Contd.)

z	V_P	V_S	ρ	g	P	K	μ
2400.0000	13.2500	7.1933	5.3403	10.1445	.1090E+12	.5691E+12	.2763E+12
2500.0000	13.3500	7.2292	5.3894	10.2139	.1145E+12	.5850E+12	.2817E+12
2600.0000	13.5000	7.2920	5.4384	10.2967	.1201E+12	.6056E+12	.2892E+12
2700.0000	13.6000	7.3276	5.4869	10.3948	.1258E+12	.6220E+12	.2946E+12
2750.0000	13.6200	7.3291	5.5116	10.4501	.1287E+12	.6277E+12	.2961E+12
2780.0000	13.6500	7.3398	5.5266	10.4854	.1305E+12	.6328E+12	.2977E+12
2800.0000	13.6400	7.3307	5.5367	10.5099	.1316E+12	.6334E+12	.2975E+12
2830.0000	13.6000	7.2000	5.5518	10.5481	.1334E+12	.6431E+12	.2878E+12
2860.0000	13.5600	7.1000	5.5668	10.5881	.1351E+12	.6494E+12	.2806E+12
2893.0000	13.5000	6.9000	5.5833	10.6343	.1371E+12	.6631E+12	.2658E+12
2893.0000	8.1000	.0000	9.8981	10.6343	.1371E+12	.6494E+12	.1809E–05
2913.0010	8.2370	.0000	9.9298	10.5905	.1392E+12	.6737E+12	.0000E+00
2932.8860	8.2730	.0000	9.9602	10.5466	.1413E+12	.6817E+12	.0000E+00
2952.6570	8.3090	.0000	9.9902	10.5026	.1434E+12	.6897E+12	.0000E+00
2972.3140	8.3430	.0000	10.0197	10.4586	.1454E+12	.6974E+12	.0000E+00
2991.8570	8.3770	.0000	10.0487	10.4145	.1475E+12	.7052E+12	.0000E+00
3011.2880	8.4110	.0000	10.0773	10.3705	.1495E+12	.7129E+12	.0000E+00
3030.6090	8.4440	.0000	10.1055	10.3264	.1515E+12	.7205E+12	.0000E+00
3049.8180	8.4760	.0000	10.1333	10.2822	.1535E+12	.7280E+12	.0000E+00
3068.9150	8.5070	.0000	10.1606	10.2381	.1555E+12	.7353E+12	.0000E+00
3087.9030	8.5380	.0000	10.1875	10.1939	.1575E+12	.7426E+12	.0000E+00
3106.7830	8.5690	.0000	10.2141	10.1498	.1594E+12	.7500E+12	.0000E+00
3125.5540	8.5980	.0000	10.2402	10.1057	.1614E+12	.7570E+12	.0000E+00
3144.2160	8.6270	.0000	10.2660	10.0615	.1633E+12	.7640E+12	.0000E+00
3162.7720	8.6560	.0000	10.2914	10.0174	.1652E+12	.7711E+12	.0000E+00
3181.2210	8.6830	.0000	10.3164	9.9733	.1671E+12	.7778E+12	.0000E+00
3199.5630	8.7110	.0000	10.3411	9.9293	.1690E+12	.7847E+12	.0000E+00
3217.8000	8.7370	.0000	10.3655	9.8852	.1709E+12	.7913E+12	.0000E+00
3235.9320	8.7630	.0000	10.3895	9.8412	.1727E+12	.7978E+12	.0000E+00
3253.9610	8.7890	.0000	10.4132	9.7972	.1746E+12	.8044E+12	.0000E+00
3271.8850	8.8130	.0000	10.4365	9.7533	.1764E+12	.8106E+12	.0000E+00
3289.7060	8.8380	.0000	10.4596	9.7095	.1782E+12	.8170E+12	.0000E+00
3307.4240	8.8610	.0000	10.4823	9.6656	.1800E+12	.8230E+12	.0000E+00
3325.0420	8.8840	.0000	10.5047	9.6219	.1818E+12	.8291E+12	.0000E+00
3342.5570	8.9070	.0000	10.5268	9.5782	.1835E+12	.8351E+12	.0000E+00
3359.9730	8.9120	.0000	10.5486	9.5345	.1853E+12	.8378E+12	.0000E+00
3377.2870	8.9290	.0000	10.5703	9.4910	.1870E+12	.8427E+12	.0000E+00
3394.5030	8.9310	.0000	10.5916	9.4475	.1888E+12	.8448E+12	.0000E+00
3411.6190	8.9430	.0000	10.6128	9.4040	.1905E+12	.8488E+12	.0000E+00
3428.6370	8.9550	.0000	10.6337	9.3607	.1922E+12	.8527E+12	.0000E+00
3445.5570	8.9650	.0000	10.6544	9.3174	.1938E+12	.8563E+12	.0000E+00
3462.3790	8.9750	.0000	10.6749	9.2742	.1955E+12	.8599E+12	.0000E+00
3479.1050	8.9850	.0000	10.6952	9.2311	.1971E+12	.8634E+12	.0000E+00
3495.7350	8.9960	.0000	10.7152	9.1880	.1988E+12	.8672E+12	.0000E+00
3512.2680	9.0070	.0000	10.7351	9.1451	.2004E+12	.8709E+12	.0000E+00

Table. (Contd.)

x	V_P	V_S	ρ	g	P	K	μ
3528.7080	9.0200	.0000	10.7547	9.1022	.2020E+12	.8750E+12	.0000E+00
3545.0520	9.0360	.0000	10.7740	9.0594	.2036E+12	.8797E+12	.0000E+00
3561.3020	9.0520	.0000	10.7932	9.0168	.2052E+12	.8844E+12	.0000E+00
3577.4600	9.0680	.0000	10.8121	8.9742	.2068E+12	.8891E+12	.0000E+00
3593.5240	9.0830	.0000	10.8308	8.9317	.2083E+12	.8935E+12	.0000E+00
3609.4950	9.0970	.0000	10.8492	8.8893	.2099E+12	.8978E+12	.0000E+00
3625.3750	9.1110	.0000	10.8675	8.8470	.2114E+12	.9021E+12	.0000E+00
3641.1630	9.1240	.0000	10.8855	8.8048	.2129E+12	.9062E+12	.0000E+00
3656.8610	9.1370	.0000	10.9033	8.7628	.2144E+12	.9103E+12	.0000E+00
3672.4690	9.1500	.0000	10.9209	8.7208	.2159E+12	.9143E+12	.0000E+00
3687.9860	9.1620	.0000	10.9383	8.6789	.2174E+12	.9182E+12	.0000E+00
3703.4140	9.1740	.0000	10.9555	8.6372	.2188E+12	.9220E+12	.0000E+00
3718.7550	9.1860	.0000	10.9725	8.5956	.2203E+12	.9259E+12	.0000E+00
3734.0060	9.1980	.0000	10.9893	8.5540	.2217E+12	.9297E+12	.0000E+00
3749.1710	9.2100	.0000	11.0059	8.5126	.2231E+12	.9336E+12	.0000E+00
3764.2460	9.2220	.0000	11.0223	8.4714	.2245E+12	.9374E+12	.0000E+00
3779.2360	9.2340	.0000	11.0385	8.4302	.2259E+12	.9412E+12	.0000E+00
3794.1410	9.2460	.0000	11.0545	8.3892	.2273E+12	.9450E+12	.0000E+00
3808.9590	9.2580	.0000	11.0704	8.3482	.2287E+12	.9488E+12	.0000E+00
3823.6920	9.2700	.0000	11.0860	8.3075	.2300E+12	.9527E+12	.0000E+00
3838.3390	9.2820	.0000	11.1015	8.2668	.2314E+12	.9565E+12	.0000E+00
3852.9030	9.2940	.0000	11.1168	8.2263	.2327E+12	.9602E+12	.0000E+00
3867.3850	9.3060	.0000	11.1319	8.1858	.2340E+12	.9640E+12	.0000E+00
3881.7810	9.3180	.0000	11.1468	8.1456	.2353E+12	.9678E+12	.0000E+00
3896.0950	9.3300	.0000	11.1615	8.1054	.2366E+12	.9716E+12	.0000E+00
3910.3270	9.3420	.0000	11.1761	8.0654	.2379E+12	.9754E+12	.0000E+00
3924.4760	9.3540	.0000	11.1905	8.0255	.2392E+12	.9791E+12	.0000E+00
3938.5460	9.3840	.0000	11.2047	7.9858	.2404E+12	.9867E+12	.0000E+00
4014.6510	9.4300	.0000	11.2758	7.7693	.2471E+12	.1003E+13	.0000E+00
4034.6510	9.4570	.5720	11.2950	7.7121	.2488E+12	.1005E+13	.3696E+10
4054.4810	9.4840	.8092	11.3140	7.6551	.2506E+12	.1008E+13	.7408E+10
4074.1420	9.5190	1.0474	11.3327	7.5985	.2523E+12	.1010E+13	.1243E+11
4093.6380	9.5330	1.1135	11.3511	7.5421	.2539E+12	.1013E+13	.1407E+11
4112.9670	9.5560	1.2295	11.3692	7.4861	.2556E+12	.1015E+13	.1719E+11
4132.1320	9.5780	1.3301	11.3871	7.4304	.2572E+12	.1018E+13	.2015E+11
4151.1350	9.5990	1.4188	11.4046	7.3751	.2588E+12	.1020E+13	.2296E+11
4169.9770	9.6190	1.4975	11.4220	7.3200	.2604E+12	.1023E+13	.2561E+11
4188.6620	9.6370	1.5631	11.4390	7.2653	.2619E+12	.1025E+13	.2795E+11
4207.1820	9.6550	1.6261	11.4558	7.2109	.2635E+12	.1028E+13	.3029E+11
4225.5490	9.6720	1.6825	11.4723	7.1569	.2650E+12	.1030E+13	.3248E+11
4243.7590	9.6880	1.7330	11.4886	7.1031	.2664E+12	.1032E+13	.3450E+11
4258.5400	9.7000	1.7688	11.5018	7.0594	.2676E+12	.1034E+13	.3599E+11
4278.5400	9.7270	1.8587	11.5194	7.0001	.2693E+12	.1037E+13	.3980E+11
4298.3510	9.7620	1.9746	11.5367	6.9413	.2708E+12	.1039E+13	.4498E+11
4317.9740	9.7770	2.0126	11.5536	6.8828	.2724E+12	.1042E+13	.4680E+11

Table. (Contd.)

z	V_P	V_S	ρ	g	P	K	μ
4337.4110	9.7990	2.0749	11.5703	6.8248	.2739E+12	.1045E+13	.4981E+11
4356.6640	9.8210	2.1356	11.5867	6.7672	.2755E+12	.1047E+13	.5284E+11
4375.7370	9.8410	2.1879	11.6029	6.7100	.2769E+12	.1050E+13	.5554E+11
4394.6260	9.8600	2.2358	11.6187	6.6532	.2784E+12	.1052E+13	.5808E+11
4413.3390	9.8780	2.2795	11.6343	6.5969	.2798E+12	.1055E+13	.6045E+11
4431.8720	9.8940	2.3161	11.6496	6.5409	.2812E+12	.1057E+13	.6249E+11
4450.2330	9.9100	2.3522	11.6646	6.4854	.2826E+12	.1060E+13	.6454E+11
4468.4160	9.9240	2.3816	11.6793	6.4303	.2840E+12	.1062E+13	.6625E+11
4486.4290	9.9370	2.4076	11.6938	6.3756	.2853E+12	.1064E+13	.6778E+11
4504.2710	9.9590	2.4609	11.7081	6.3214	.2867E+12	.1067E+13	.7090E+11
4522.6600	9.9700	2.4796	11.7226	6.2653	.2880E+12	.1069E+13	.7208E+11
4542.6600	9.9930	2.5325	11.7382	6.2043	.2895E+12	.1072E+13	.7528E+11
4562.4460	10.0240	2.6077	11.7535	6.1438	.2909E+12	.1074E+13	.7993E+11
4579.7300	10.0500	2.6686	11.7668	6.0909	.2921E+12	.1077E+13	.8380E+11
4599.7300	10.0910	2.7675	11.7820	6.0295	.2936E+12	.1079E+13	.9024E+11
4619.5090	10.1300	2.8581	11.7969	5.9687	.2950E+12	.1082E+13	.9637E+11
4639.0620	10.1670	2.9410	11.8114	5.9086	.2963E+12	.1085E+13	.1022E+12
4658.4020	10.1820	2.9658	11.8257	5.8489	.2977E+12	.1087E+13	.1040E+12
4677.5240	10.2250	3.0614	11.8396	5.7899	.2990E+12	.1090E+13	.1110E+12
4696.4300	10.2560	3.1250	11.8532	5.7315	.3002E+12	.1092E+13	.1158E+12
4715.1290	10.2850	3.1828	11.8666	5.6736	.3015E+12	.1095E+13	.1202E+12
4733.6150	10.3120	3.2350	11.8797	5.6163	.3027E+12	.1097E+13	.1243E+12
4751.8990	10.3370	3.2818	11.8925	5.5596	.3039E+12	.1100E+13	.1281E+12
4769.9750	10.3600	3.3234	11.9050	5.5035	.3051E+12	.1102E+13	.1315E+12
4777.9620	10.3700	3.3413	11.9105	5.4787	.3057E+12	.1104E+13	.1330E+12
4797.9620	10.4120	3.4245	11.9241	5.4165	.3069E+12	.1106E+13	.1398E+12
4817.7110	10.4510	3.4994	11.9374	5.3550	.3082E+12	.1109E+13	.1462E+12
4837.2120	10.4880	3.5686	11.9504	5.2943	.3094E+12	.1112E+13	.1522E+12
4856.4680	10.5220	3.6304	11.9630	5.2343	.3106E+12	.1114E+13	.1577E+12
4867.1720	10.5400	3.6623	11.9701	5.2009	.3113E+12	.1116E+13	.1605E+12
4887.1720	10.5920	3.7604	11.9829	5.1385	.3125E+12	.1118E+13	.1694E+12
4906.9060	10.6410	3.8502	11.9955	5.0769	.3137E+12	.1121E+13	.1778E+12
4923.6400	10.6800	3.9199	12.0060	5.0246	.3148E+12	.1123E+13	.1845E+12
4943.6400	10.7430	4.0346	12.0184	4.9621	.3160E+12	.1126E+13	.1956E+12
4963.3650	10.8020	4.1391	12.0305	4.9005	.3171E+12	.1129E+13	.2061E+12
4983.6400	10.8600	4.2393	12.0427	4.8372	.3183E+12	.1132E+13	.2164E+12
5000.0000	9.7000	.0000	12.0525	4.7861	.3192E+12	.1134E+13	.0000E+00
5197.3220	10.1500	.0000	12.1304	4.1754	.3292E+12	.1250E+13	.0000E+00
5197.3220	10.6400	3.3250	12.6700	4.1754	.3292E+12	1248E+13	.1401E+12
5212.6120	10.6500	3.3281	12.6779	4.1218	.3300E+12	.1251E+13	.1404E+12
5357.3300	10.9100	3.4094	12.7295	3.6108	.3367E+12	.1318E+13	.1480E+12
5393.2520	10.9600	3.4250	12.7439	3.4832	.3383E+12	.1331E+13	.1495E+12
5456.2120	11.0300	3.4469	12.7658	3.2584	.3409E+12	.1351E+13	.1517E+12
5505.6800	11.0800	3.4625	12.7823	3.0809	.3428E+12	.1365E+13	.1532E+12
5925.5600	11.1200	3.4750	12.8029	1.5458	.3512E+12	.1377E+13	.1546E+12
6371.0300	11.2000	3.5000	12.8004	.0000	.3512E+12	.1397E+13	.1568E+12

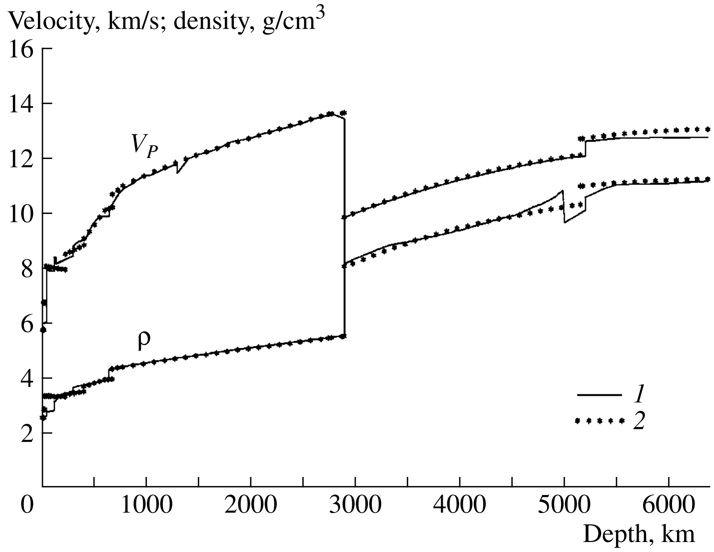

Fig. 7. Velocity and density distributions in two models of the Earth: (*1*) this work; (*2*) PREM.

which means a nonvanishing *S* wave velocity there. As can be seen from Fig. 5, the *S* velocity monotonically increases from zero to 4.2 km/s in the depth range 400–500 km.

The question of which of these of two models is preferable remains open. To answer this question unambiguously, it is necessary to invoke other data, in particular, data on the Earth's free vibrations.

Here, we discuss once again the validity of applying the Williamson–Adams equation to media with a varying chemical composition. Figure 6 presents two curves, one of which shows the variation in $\frac{g}{\Phi}$ with depth and the other plots the difference $\delta = \beta t$ between $\frac{g}{\Phi}$ and $\frac{1}{\rho}\frac{d\rho}{dz}$. The absolute value of δ averages $\approx 10^{-6}$ km^{-1} and exceeds 3×10^{-6} km^{-1} only at some points. The quantity $\frac{g}{\Phi}$ varies from 10^{-4} km^{-1} in the outer core to 3×10^{-4} km^{-1} in the upper mantle. The difference of at least two orders of magnitude indicates that the Williamson–Adams equation can be used to determine the density in the Earth's interior at all depths.

To obtain the density values in the C layer, Bullen approximated the density distribution in this layer by a quadratic parabola. The results of the determination of $\rho(z)$ in the mantle show that the density distribution curves of the standard model and those obtained in the present work are very similar for all layers, including the transitional C layer. In addition, this indicates that, in practice, the Williamson–Adams equations are also applicable to elastic media with a chemical composition varying with depth.

Figure 7 shows the density distributions with depth in the PREM model [Dziewonski and Anderson, 1981] and in the model obtained in the present work. As seen from the figure, these curves coincide virtually everywhere except for the inner core and the uppermost part of the Earth, where the difference between the curves is due to the different velocity curves. In the Earth's inner core, the PREM density curve lies above the curve obtained in this work and the PREM density gradient is higher.

The relative errors in *M* and *J* calculated from the density distribution obtained in this work are $\Delta M/M = -8.682 \times 10^{-5}\%$ and $\Delta J/J = 8.216 \times 10^{-5}\%$, respectively; this considerably exceeds the uncertainties in the observed values.

CONCLUSIONS

A new pattern of the depth distribution of seismic velocities in the Earth entails a revision of the distributions of density and elastic parameters. The density distribution is determined with the use of the well-known Williamson–Adams relation, which, as is shown in this paper, can be applied to media whose chemical composition can either vary with depth or be constant. The validity of this is illustrated by the numerical modeling, which showed in addition that rocks composing deeper layers can have, under normal conditions, lower densities compared to rocks of the overlying layers. This difference can be fairly considerable.

A distinctive feature of the density distribution derived from the new velocity curve is that it differs little from the PREM distribution, obtained from data on the Earth's free vibrations. The fact that the two approaches to the determination of the Earth's density distribution yield very similar results indicates that we have actually obtained additional relations for determining physical parameters in the Earth's interior.

As regards the elastic parameters k and μ, their behavior in the Earth's core can be different depending on the physical processes in the core. In particular, the bulk modulus can have a negative jump at the outer–inner core boundary, and the shear modulus can be nonzero in the lower part of the outer core. To remove this ambiguity, additional geophysical data are required.

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