Izvestiya, Physics of the Solid Earth, Vol. 40, No. 6, 2004, pp. 477–490. Translated from Fizika Zemli, No. 6, 2004, pp. 26–41.
Original Russian Text Copyright © 2004 by Burmin.
English Translation Copyright © 2004 by MAIK "Nauka/Interperiodica" (Russia).

Seismic Wave Velocities in the Earth's Core

V. Yu. Burmin
Schmidt United Institute of Physics of the Earth, Russian Academy of Sciences,
Bol'shaya Gruzinskaya ul. 10, Moscow, 123995 Russia
e-mail: burmin@ifz.ru
Received August 21, 2002

Abstract—Using record sections of seismic waves from deep earthquakes recorded by the worldwide network, traveltime curves are constructed for refracted and reflected short-period longitudinal waves traveling in the Earth's core. These curves are used to obtain a velocity structure of the core that is in good agreement with observed data and provides an explanation for the origin of the so-called precursors (waves recorded at the Earth's surface as first arrivals at epicentral distances of 134°–142°). The velocity structure in the outer core is characterized by a ~500-km thick layer with a high positive gradient of velocity present in the lower part resting on a lower velocity layer up to 200 km in thickness (the F zone). The velocity in the inner core first increases rather sharply to a depth of about 5500 km, after which its increase toward the center of the Earth becomes nearly linear, with a gradient slightly higher than is predicted by standard models.

INTRODUCTION

There is no doubt that the Earth's core has a significant, often determining, effect on many global processes observable on the Earth's surface. The existence of the Earth's core was noted for the first time by Oldham [1906] in his study of earthquake records obtained near anticenters. He discovered the presence of a velocity jump at approximately the middle of the Earth's radius. In 1914, when a general velocity structure of the Earth had been constructed from *P* waves, Gutenberg [1963] demonstrated the *P* wave velocity drops from 13.5 to 8.5 km/s at a depth of about 2900 km. The region below this depth was called the Earth's core. As was also established, transverse waves do not cross the core.

In the 1930s, analysis of seismograms revealed the presence of waves that could not be interpreted in terms of the hypothesis of a purely liquid core. Lehmann supposed in 1936 that the core consists of outer and inner parts and that the velocity in the inner core is higher. This was confirmed by Jeffreys [1939a], who gave the classic velocity cross section of the core (Fig. 1), based on the traveltime branches of the refractions *PKP*1 and *PKP*2, differing in slope.

We discuss in more detail the technique by which the velocity distribution was obtained in the upper part of the outer core. The point is that the mantle refraction traveltime curve ends at epicentral distances at which the waves reach the mantle/core boundary. This is due to the fact that, in the upper part of the outer core, there exists a layer in which the ratio $v(r)/r$ is smaller than at the mantle base. As is known, the problem of velocity determination from traveltime data does not have a unique solution in this case. The velocity distribution in this layer was obtained by using the procedure proposed by Wadati and Masuda [1934], which can be described as follows. A *P* wave striking an interface between two media differing in their elastic characteristics at each point of the interface experiences both refraction and reflection. In this case, seismic rays along these waves' travel have the same ray parameter. Subtracting the reflection traveltime from the refraction traveltime, we obtain the traveltime of the wave propagating beneath the interface. Applying this procedure to each point of the observed refraction traveltime curve and taking migration into account, we can obtain the

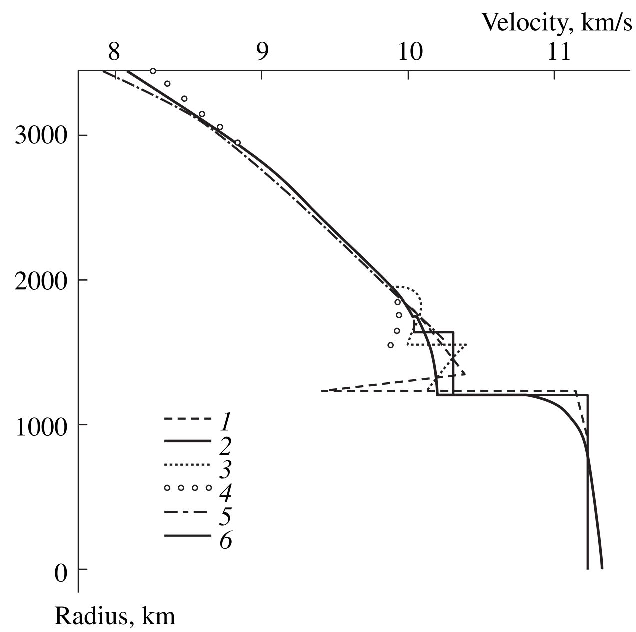

Fig. 1. Distributions of the *P* wave velocity in the core from data of various authors Qamar [1973]: (*1*) Jeffreys [1939]; (*2*) Bolt [1962]; (*3*) Adams and Randall [1964]; (*4*) Randall [1970]; (*5*) Hales and Roberts [1971]; (*6*) KOR5 model.

traveltime curve of the wave traveling beneath the interface. If the latter is the mantle/core boundary, this procedure yields the traveltime curve of the wave traveling inside the Earth, as if observations were conducted at the core boundary. Of course, the shadow zone is present in this case as well, and the traveltime curve starts not from zero but from a certain point in the epicentral distance–time plane. In order to find the velocity distribution in the shadow zone, the converted traveltime curve is complemented by a convex curve connecting the first point of the curve with its zero point. Evidently, the number of curves that can be constructed in this way is infinite and, accordingly, it seems that one can obtain an infinite set of velocity curves providing a monotonically increasing ratio $v(r)/r$. However, this is not true. As was shown in [Geiko, 1982; Burmin, 1996], there exists a unique monotonic velocity curve, which is the solution in the waveguide. Therefore, traveltime curve points cannot be connected in an arbitrary way, and not every (in this case convex) curve is a traveltime curve.

The velocity distribution, obtained by Jeffreys, includes a transitional outer–inner core layer in the range $0.36R < r < 0.40R$, i.e., between the depths 4980 and 5120 km (Fig. 1). Afterward, this layer was called the F zone. Jeffreys introduced a negative P velocity gradient equal to $dv_P/dz = -0.0075$ s^{-1} in this part of the core and a jump in v_P from 9.40 to 11.16 km/s at the lower boundary of the F zone. Gutenberg and Richter [1938, 1939] obtained a v_P distribution that was continuous throughout the core but had large positive gradients in a certain depth interval approximately coinciding with the F zone. In this connection, Jeffreys [1960, p. 118] noted that Gutenberg and Richter calculated the velocity distribution in the core yielding no decrease in velocity near the inner core, but he did not understand as yet how such a distribution could be brought into agreement with the length of the reversed branch.

Data gathered from the 1950s onward yielded evidence that, at epicentral distances from 125° to 140°, the *PKP* waves are preceded by the so-called precursors having periods of 0.5–1.0 s and arriving about 10–20 s earlier than the *PKP*1 waves with periods of 2 s and more [Gutenberg, 1953, 1958a]. The precursors have rather small amplitudes and therefore can easily be missed in the preprocessing of seismograms. Gutenberg [1953, 1958a, 1958b] supposed that the precursors are a consequence of dispersion in the transition zone, where short-period waves are faster than long-period ones. The main argument against this hypothesis is the fact that, according to observations, wave periods do not gradually increase from the onset of a short-period precursor to the *PKP*1 arrival.

Using traveltime data obtained from records of deep earthquakes in the region of the Fiji, New Hebrides, and Celebes islands, Hai [1963] obtained a velocity distribution appreciably differing from the Jeffreys distribution in the transition zone extending from a depth of about 4400 km to the inner core boundary, whose radius was set equal to 1236 km. The velocity v_P was supposed to be continuous in the outer core and the transition zone. In the depth interval 4380–4480 km, v_P first decreased from 9.91 to 9.86 km/s and then increased to 9.98 km/s. At the inner core boundary, the velocity jumped from 10.27 to 10.80 km/s, after which its behavior was governed by the Gutenberg distribution in the inner core, giving $v_P = 11.24$ km/s at the center.

Bolt [1962, 1964] supposed that the presence of precursors can be explained in terms of the ray-path theory by introducing an additional traveltime branch at distances from 124.0° to 160.0° and the related transition layer with two velocity jumps (at depths near 5000 km), whereas the remaining distribution is the same as in the Jeffreys model (Fig. 1).

Adams and Randall [1963, 1964] obtained additional evidence supporting the existence of an extra traveltime branch in the range 130.0°–156.5°. To interpret this branch, these authors introduced one more velocity jump located above the inner core boundary (at $r = 1980$ km). Thus, they proposed a model with two transition layers between the outer and inner core (Fig. 1).

Randall [1970] proposed a different P velocity distribution in the outer core. The velocity in the upper part of his outer core model is 8.26 km/s, whereas the velocity in the lower part (4450–4800 km) decreases from 9.946 to 9.890 km/s (Fig. 1).

A basic change of the Jeffreys model was the removal of the negative velocity gradient in the F zone. The absence of large negative gradients of velocity throughout the core was supported by the majority of later studies. Due to its parametric simplicity, the Bolt model was often used for the density estimation in the inner core in the period from 1964 to 1970.

In the 1970s, results of the interpretation of precursors raised much doubt, because the ray-path theory, used at that time, failed to yield a reasonable velocity distribution in the core and to interpret the observed curvature of precursor traveltime curves (T, θ). Exhausting the potential of the ray-path theory, Cleary and Haddon [1972] were compelled to conclude that precursors should be related to the scattering of *PKP* waves by a caustic near the mantle/core boundary in the D'' zone, rather than to the presence of transition layers of the Bolt type or to discontinuities of the velocity curve in the outer or inner core. These authors showed that the observed traveltime curves and apparent velocities of precursors agree with the theoretical curves obtained for primary scattered waves. Doornbos and Vlaar [1973] supported the hypothesis of Cleary and Haddon on the scattering of *PKP* waves by caustics, but they supposed that the scattering takes place at various depths in the lower mantle.

However, we should note that the mechanism of wave scattering by caustics was afterward replaced with the scattering by random small-scale heterogene-

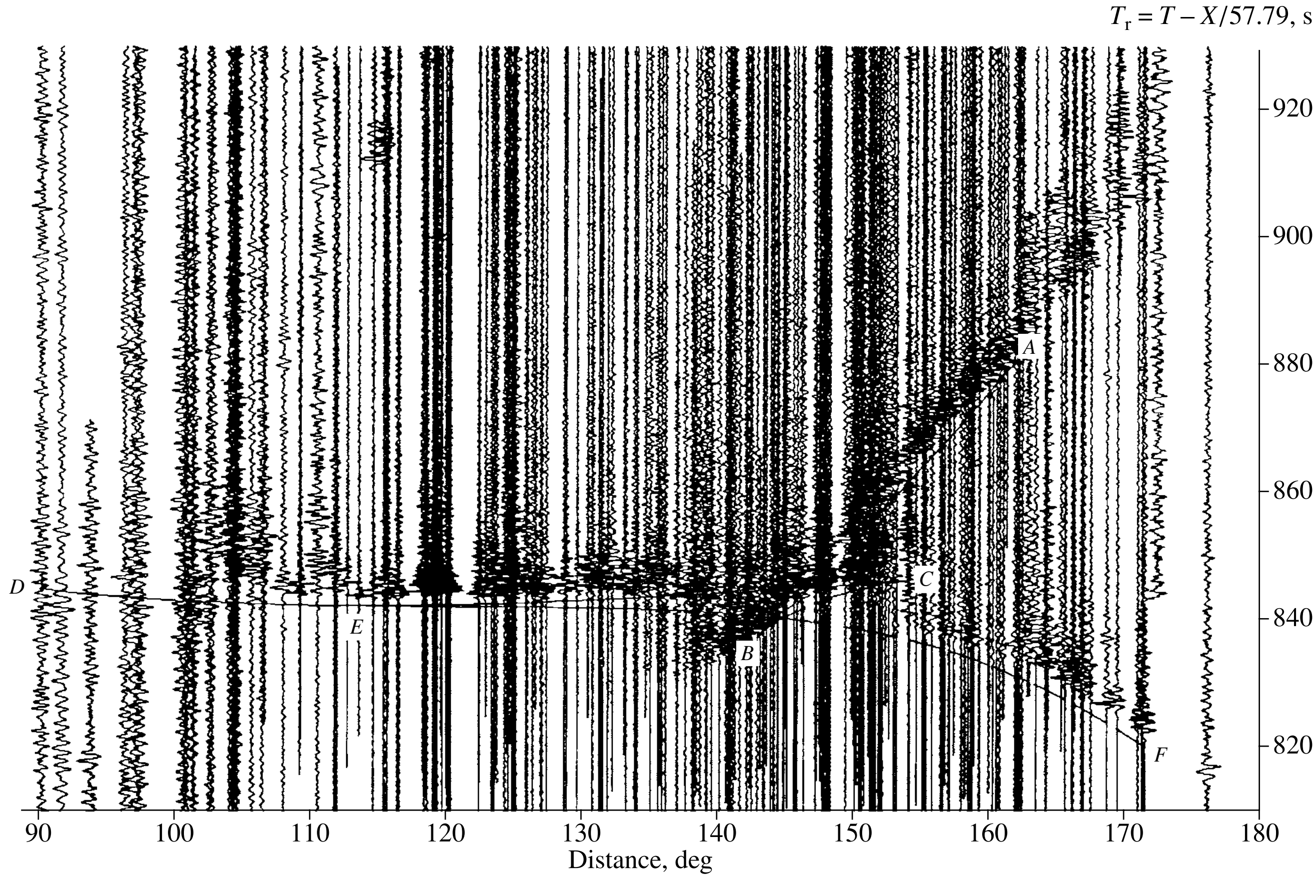

Fig. 2. Record section and theoretical traveltime curves of the IASPEI91 model.

ities and has not been mentioned by anybody since then. One of the recent works devoted to this subject is the study by Hedlin and Shearer [2002]. In any case, irrespective of the scattering mechanism of seismic waves in the lower mantle, the scattering wave traveltime curve should be symmetric on the Earth's surface and the wave amplitude at its minimum point should be highest. Nothing of this type is observed in seismic record sections (Figs. 2–5), and the record sections of synthetic seismograms of scattered and regular waves in the range of epicentral distances under study have not been published.

The *PKP* traveltime curve is difficult to construct because, in the interval 120°–160°, different *PKP* phases arrive with small delays relative to one another. Therefore, it is difficult to extract them from a seismogram and especially to identify these phases in order to associate them with a specific traveltime branch. In this respect, attempts were made to revise the very approach to the determination of the traveltime curve in a transition zone. Azbel' and Yanovskaya [1975] proposed a method according to which data on the arrival times of all successive phases are jointly taken into account and the solution as a whole, rather than separate branches, is determined. In this way, a stochastic model of the data in use was actually determined and their distribution function was constructed, after which a stochastic method was applied to the estimation of parameters. The resulting traveltime plot included more than four branches, and the velocity cross section contained numerous jumps and lower velocity zones.

Constructing the PREM model, Dziewonski and Anderson [1981] proceeded from the hypothesis that the core consists of two homogeneous parts.

The recently widely acknowledged IASPEI91 model [Kennett, 1992] was also constructed on the basis of the hypothesis of a core that consists of two homogeneous parts, with the velocity jump Δv_P = 0.84 km/s at their interface at the depth z = 5153.9 km.

We mentioned here only a small part of publications devoted to the structure of the Earth's core. A more complete idea of the diversity of approaches and concepts can be gained, in addition to the aforementioned studies, from [Antonova, 1971; Bullen, 1975; Yanovskaya, 1975; Ergin, 1967; Morelli *et al.*, 1986; Stixrude and Brown, 1970; Song and Helmberger, 1998; and others]. The structure of the Earth's core is discussed in detail in [Jacobs, 1976]. However, the problem of the core's structure has not been completely solved.

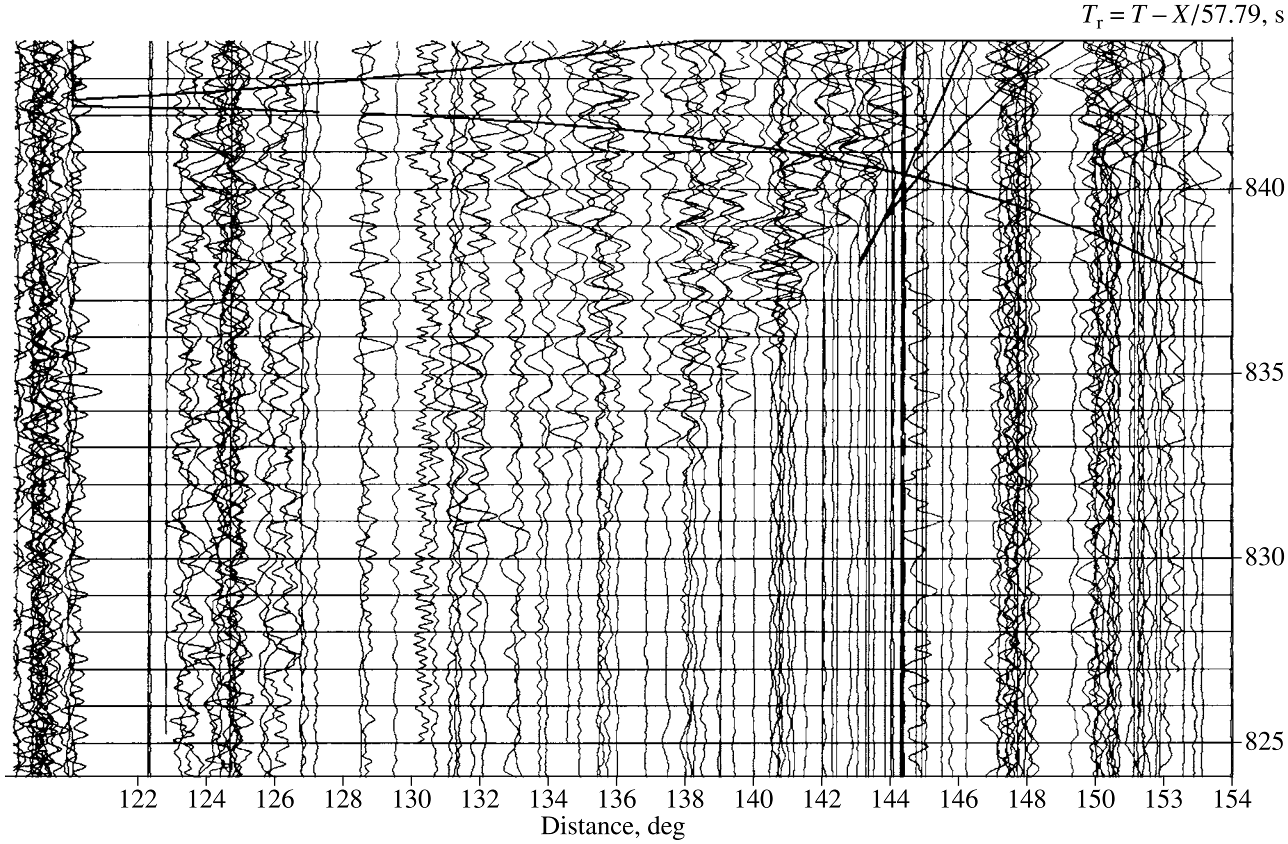

Fig. 3. precursors and theoretical traveltime curves of the IASPEI91 model.

The P wave velocities in the inner core have been determined more or less satisfactorily. Their velocity is nearly constant in the majority of models, except for the models of Buchbinder [1971] and Qamar [1973], in which a pronounced velocity gradient exists in the upper part of the inner core. The Qamar model KOR5 is shown in Fig. 1.

Summarizing the above review, we may state that the seismic wave velocity distribution in the outer core is still unclear. In particular, estimates of P wave velocities immediately under the mantle/core boundary differ within wide limits: from 7.9 km/s [Hales and Roberts, 1971] to 8.26 km/s [Randall, 1970].

The problem of the velocity distribution in the inner/outer core transition layer (F zone) remains actually unsolved. Numerous models varying in the number of layers exist for this zone.

Based on the theory previously developed by the author [Burmin, 1993] for the inversion of discontinuous refraction and reflection traveltime data specified on a discrete set of points, the present paper is intended for the construction of a velocity model in the Earth's core providing the best agreement with data of seismic observations. The solution of this problem is based on the interpretation of a large number of short-period records of P waves from deep earthquakes and the application of the traveltime inversion theory, mentioned above.

Before solving the above problem, it is appropriate to compare the theoretical traveltime curves of refracted and reflected waves derived from a standard model (e.g., IASPEI91) with record sections in order to gain insights into the extent of agreement between modern models and the actual distribution of seismic velocities in the Earth's core. This analysis is presented below.

INITIAL DATA

In order to solve the problem formulated above, we used experimental data containing records of seismic waves that crossed the Earth's core and enabling the construction of the related traveltime curves suitable for interpretation. Presently, stations of the worldwide seismic network record all large events whose wave patterns are helpful for the study of the core's velocity structure. We used digital data of the National Earthquake Information Center of the US Geological Service obtained at stations of the worldwide network in the period from 1980 through 1988. These data include records of long-, medium-, and short-period seismographs. Only the records of short-period seismographs

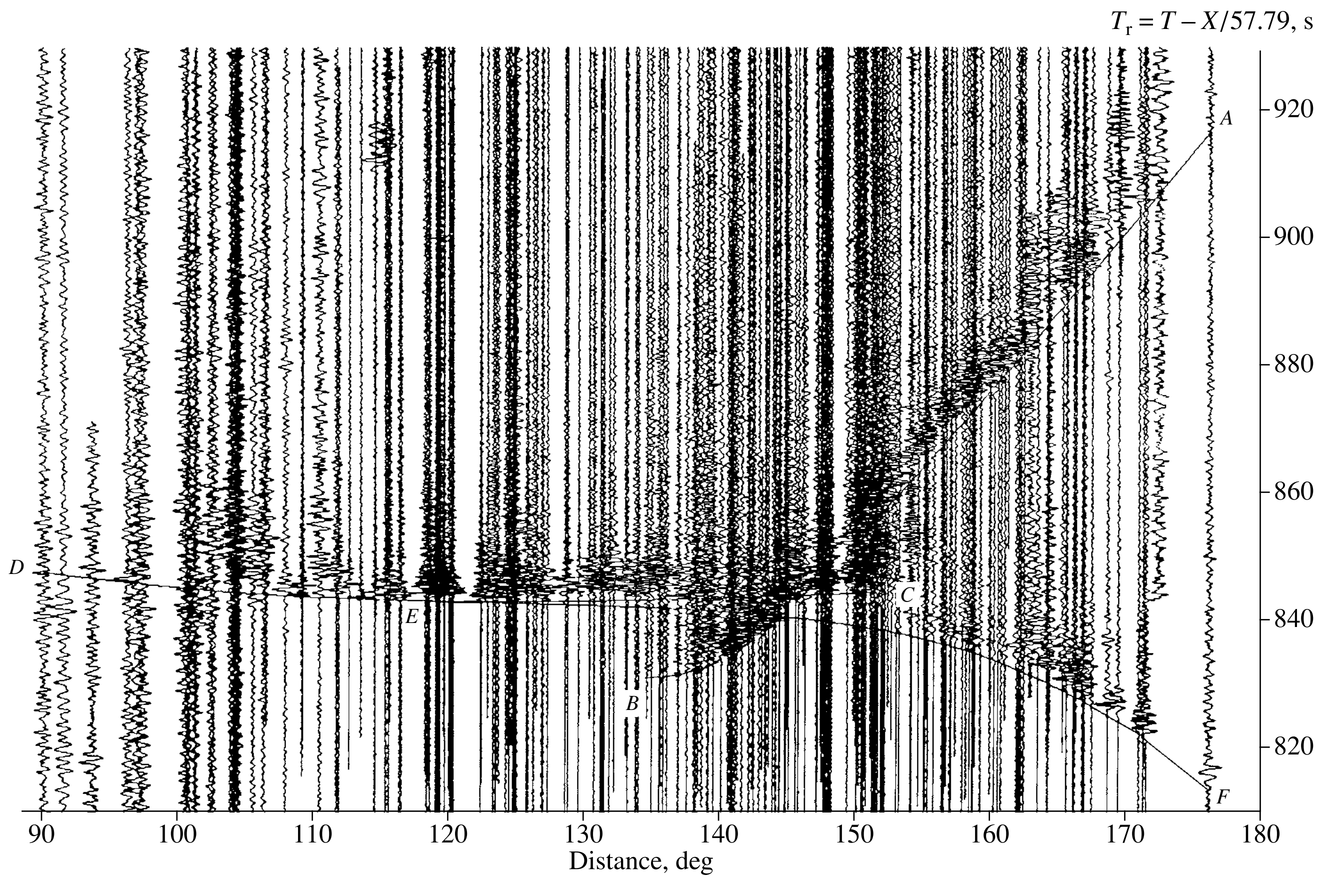

Fig. 4. Record section and theoretical traveltime curves of the new velocity model.

(the passband 0.1–6.0 Hz) were used for interpretation because of higher resolution of these instruments.

Records of seismic waves from all earthquakes deeper than 100 km with magnitudes greater than 5.5 were inspected. Records with the most distinct signals, preferably from earthquakes deeper than 500 km, were selected. Shallower earthquakes were included only if gaps in record sections had to be filled. Although the volume of initial data was large (more than 1000 seismograms), we failed to select an adequate number of records at epicentral distances of 172° to 180°. In this range, only one record at 176° was available.

All records were filtered in a frequency range of 0.5–5.0 Hz and were reduced to a fixed source at a depth of 500 km with due regard for traveltime differences and migration. The data were represented as a record section of the vertical component of seismographs in the reduction to 57.79 km/s in the ranges of epicentral distances 90°–180° and times 810–930 s (Figs. 2 and 3). Moreover, each record was normalized to the maximum wave amplitude of the trace. In all, the record section contains 263 records.

SOLUTION OF THE FORWARD KINEMATIC PROBLEM OF SEISMICS FOR A SPHERICALLY SYMMETRIC EARTH

In order to test the correctness of a chosen velocity model and to compare various models, theoretical traveltime curves should be calculated for various types of waves. This forward kinematic problem is a constituent of the mathematical modeling of seismic wave propagation in the Earth. The program 'dir1.exe' developed by the author was used for calculating traveltime curves of refractions, head waves, and primary reflections in a spherically symmetric elastic medium (with the elastic wave velocity depending solely on radius). In this case, the so-called two-point problem is solved, i.e., a problem in which a seismic ray connects two given points of the medium. The solution algorithm is briefly described below.

The following assumptions are used: (1) the Earth is a sphere of radius R, and (2) the seismic wave velocity is a piecewise twice differentiable function of radius $v = v(r)$ limited in the entire depth interval $[0, R]$.

In what follows, we distinguish four types of waves traveling in the Earth:

waves traveling upward from a source;

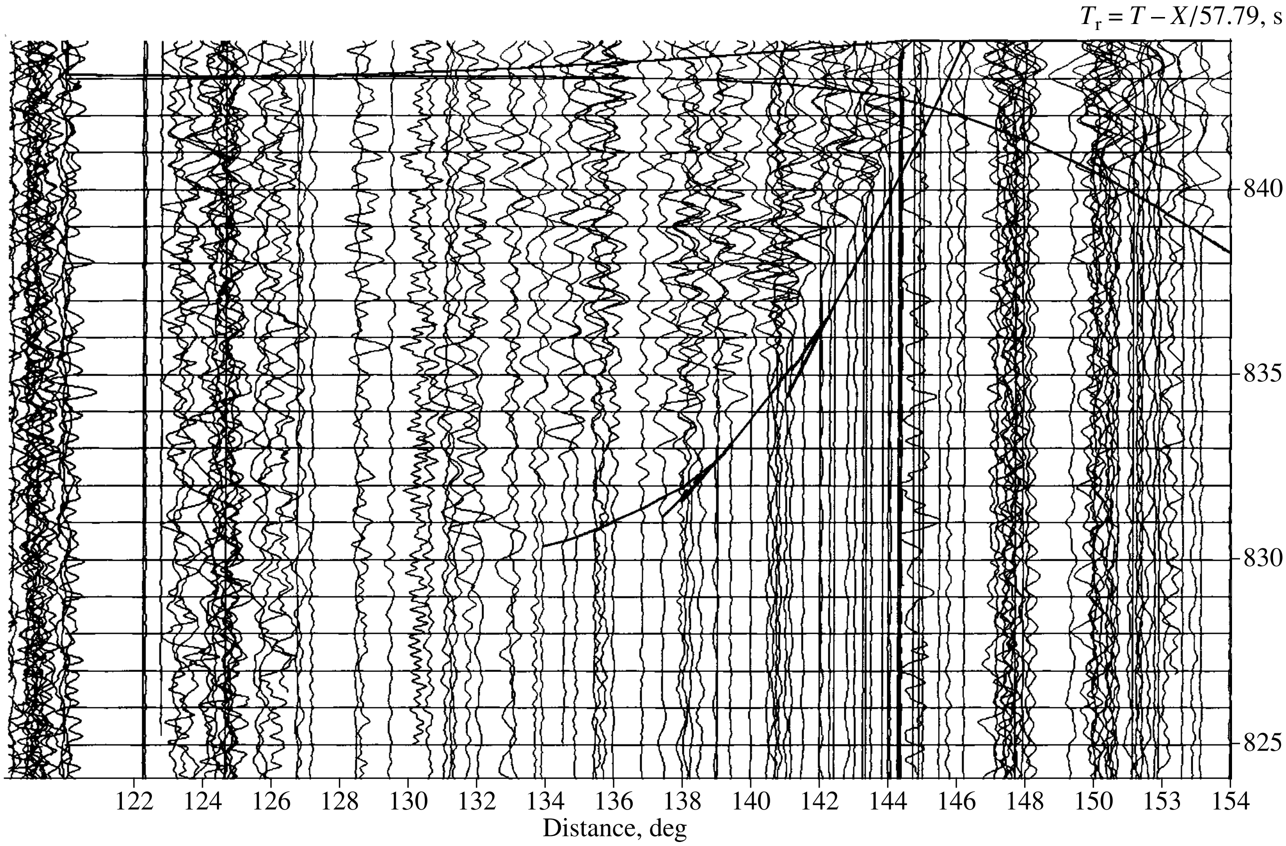

Fig. 5. precursors and theoretical traveltime curves of the new velocity model.

refracted waves traveling along rays that differ in maximum penetration depth $z_M = R - r_M$. Given refracted rays striking the Earth's surface at angles φ, $\psi \in \left[0, \frac{\pi}{2}\right]$ measured from the vertical, the relation $\varphi < \psi$ implies $z(\varphi) > z(\psi)$, or $r(\varphi) < r(\psi)$;

head waves reaching the Earth's surface at the same angle i and produced by waves traveling along an interface;

primary reflections associated with velocity jumps in an elastic medium at depths $z = z_i^*$ $(r = r_i^*)$ $(i = 1, 2, \ldots, n)$. Given a reflection from the boundary $z = z_j^*$ $(r = r_j^*)$, the relation $z(\varphi) = z^*$ $(r(\varphi) = r^*)$ is valid for all rays with emergence angles $\varphi \in [0, \varphi_1]$, where φ_1 is the angle of the critical ray.

We consider an elastic disturbance traveling in a great circle cross section of the sphere, with the origin of the polar coordinate system (θ, r) placed at the center of this circle (θ is the angular distance, or the arc length measured in angular units, and r is the distance in linear units of measurement from the center to an inner point of the circle). Let a source of elastic waves be placed at a point $(0, R)$ on the boundary of the circle and let the waves be recorded also on the Earth's surface at points (θ, R).

Since the seismic ray originates on the Earth's surface, it has a maximum penetration point (θ_m, r_m). In the refraction case, the relations $\frac{\partial t}{\partial \theta} = \frac{r_m}{v(r_m)} = p$ and $r_m = r(p)$ are valid at this point. Here, p is the ray parameter. In the reflection case, we have $r_m = r^*$, where r^* is the distance from the Earth's center to the reflector.

Equations of refraction and reflection traveltime curves have the following parametric form [Savarenskii and Kirnos, 1955]:

$$\theta(p) = 2p\int_{r_m}^{R} \frac{1}{\sqrt{[r/v(r)]^2 - p^2}}\frac{dr}{r},$$
$$t(p) = 2\int_{r_m}^{R} \frac{[r/v(r)]^2}{\sqrt{[r/v(r)]^2 - p^2}}\frac{dr}{r}. \quad (1)$$

The head wave traveltime curve can be written in an explicit form:

$$t(p, \theta) = 2\int_{r_m}^{R} \sqrt{\frac{r^2}{v^2(z)} - p^2}\frac{dr}{r} + p\theta, \quad \theta_s \le \theta \le \infty, \quad (2)$$

where θ_s is the coordinate of the initial point of the curve and p = const for all $\theta \in [\theta_s, \infty)$.

If the source is located at a depth z^*, the traveltime equation of a wave traveling upward from the source has the form

$$\theta_1(p) = p\int_{r^*}^{R} \frac{1}{\sqrt{[r/v(r)]^2 - p^2}}\frac{dr}{r},$$
$$t_1(p) = \int_{r^*}^{R} \frac{[r/v(r)]^2}{\sqrt{[r/v(r)]^2 - p^2}}\frac{dr}{r}, \qquad (3)$$

where r^* is the distance from the Earth's center to the source and $0 \le p \le r/v(r^*)$.

The traveltime equation of a refraction traveling downward from a source located at a depth z^* is written as

$$\theta_2(p) = 2p\int_{r_m}^{r^*} \frac{1}{\sqrt{[r/v(r)]^2 - p^2}}\frac{dr}{r} + \theta_1(p),$$
$$t_2(p) = 2\int_{r_m}^{r^*} \frac{[r/v(r)]^2}{\sqrt{[r/v(r)]^2 - p^2}}\frac{dr}{r} + t_1(p), \qquad (3')$$

where $r^* = R - z^*$ and $\frac{r}{v(r_m)} \le p \le \frac{r}{v(r^*)}$.

The complete traveltime curve of the seismic wave traveling from a deep source is convex downward in the interval $[0, \theta^*]$ and convex upward in the interval $[\theta^*, \theta > \theta^*]$. The point $(0, t_0)$ corresponds to a ray departing from the source vertically ($p = 0$). The traveltime point $M^* = (\theta^*, t^*)$ corresponds to a ray departing from the source at an angle $\pi/2$ with respect to r. At the point M^* $p = r/v(z^*)$ and the second derivative of the traveltime curve changes its sign; i.e., M^* is an inflection point of the curve.

In order to calculate the traveltime curve, the velocity cross section is represented as a column, i.e., a set of layers, each specified by its thickness (or by the depth to its upper boundary) and velocity values at its upper and lower boundaries (or above and under the corresponding boundary). Given an approximate velocity cross section, integrals (1)–(3) and (3') can be calculated with the use of explicit formulas expressed through trigonometric, inverse trigonometric, and logarithmic functions. In our computer program, each layer of the velocity column is divided into N elementary layers characterized by constant velocities and small thicknesses. In this case, the integrals are replaced by sums, and all calculations are simplified. Such an approach increases, to an extent, the computation time, but the accuracy of the calculations depends solely on the division step of the velocity column and does not depend on the accuracy of calculation of trigonometric and inverse trigonometric functions if they are used in the explicit formulas. A numerical experiment showed that a 1-km step of division is quite adequate.

KINEMATIC INVERSION OF SEISMIC DATA IN TERMS OF A SPHERICALLY SYMMETRIC EARTH

1. Inversion of a Continuous Refraction Traveltime Curve

The inverse kinematic problem of seismics was solved for the first time in the early 20th century by G. Herglotz, E. Wiechert, and G. Bateman in the case of a spherically symmetric Earth with a surface source. They showed that, if the function $u(r) = r/v(r) > 0$ is a monotonic increasing function, where $v(r)$ is the seismic velocity at a distance r from the Earth's center, then $v(r)$ is uniquely determined from the traveltime curve of a seismic wave observed at the Earth's surface. The function $v(r)$ is determined by the seismic traveltime inversion formula [Savarenskii and Kirnos, 1955]

$$\ln\frac{R}{r_1} = \frac{1}{\pi}\int_0^{\theta_1} \operatorname{arccosech}\left[\frac{p(\theta)}{p(\theta_1)}\right]d\theta, \qquad (4)$$

where R is the Earth's radius, r_1 is the radius of the apex of the ray emerging at a distance θ_1, and $p(\theta)$ is the current value of the ray parameter ($\theta_1 \ge \theta \ge 0$). Formula (4) and the relation $p(\theta_1) = \frac{r_1}{v(r_1)} = dt/d\theta_{\theta = \theta_1}$ determine the seismic velocity as a function of radius r.

2. Inversion of a Discontinuous Refraction Traveltime Curve

Formula (4) was obtained for a continuous traveltime curve and a spherically symmetric Earth. The solution of this problem has not been considered in the case of a discontinuous traveltime curve. In this respect, we solve the inverse problem here for a plane surface of the Earth with the subsequent transformation of a half-plane into a circle using the formula [Gerver and Markushevich, 1963]

$$\theta = x/R, \quad r = R\exp\left[-\frac{z}{R}\right],$$
$$v(r) = v(z)\exp\left[-\frac{z}{R}\right]. \qquad (5)$$

The traveltime of the seismic wave remains constant here. In this case, the inverse transformation has the form

$$x = \theta R, \quad z = R\ln\left[\frac{R}{r}\right], \quad v(z) = v(r)\frac{R}{r}. \qquad (5')$$

Formulas for the inversion of a discontinuous traveltime curve in the case of a plane Earth were obtained in [Burmin, 1980b, 1996].

Refracted waves are represented by rays with various maximum penetration depths z_m. The refraction and reflection traveltime equations for a plane Earth have the following form:

$$x(p) = 2p\int_0^{z_m}\frac{dz}{\sqrt{v(z)^{-2}-p^2}},$$
$$t(p) = 2\int_0^{z_m}\frac{dz}{v^2(z)\sqrt{v^{-2}(z)-p^2}}. \quad (6)$$

The inversion formula of the refraction traveltime curve has the form

$$z(q) = z_m = z^* + \frac{1}{\pi}\int_q^{u^*}\frac{x(p,u^*)}{\sqrt{p^2-q^2}}dp,$$

where $x(p, u^*) = x(p) - 2p\int_0^{z^*}\frac{d\eta}{\sqrt{v^{-2}(\eta)-p^2}}$, $q = u(z_m) \le p \le u(z^*) = u^*$, $z^* \le z_m \le z_M$, $u(z) = v^{-1}(z)$, z_M is the maximum penetration depth of seismic rays corresponding to the last point of the traveltime curve, and $x(p, u^*)$ is the p-dependent distance between the emergence point of the seismic ray penetrating to the depth $z_m > z^*$ and its entrance point at the depth $z = z^*$.

The formula for the refraction inversion determines the maximum penetration depth of the ray emerging at a distance $x(p)$ from the source. Along with the relation

$$v^{-1}(z_m) = u(z_m) = t'[x(q)],$$

determining the seismic velocity at the depth $z_m = z(q)$, the maximum penetration depth provides the sought-for dependence $v = v(z)$ in a parametric form for $z^* \le z_m \le z_M$.

Discontinuities of refraction traveltime curves are known to indicate the presence of waveguides or constant-velocity layers [Burmin, 1993]. In order to use the inversion formula, one should know the velocity distribution in the interval $[0, z^*]$ and, in particular, in waveguides above the point z^*.

In the general case, a refraction traveltime curve is usable for determining the functions $\lambda(u) = -dz/du \ge 0$, $z \in [z^*, z_M]$, and $H(u) = \text{mes}\{z : z \in [z^*, \bar{z}^*], v^{-1}(z) \le u\}$ from the solution of the two Fredholm integral equations of the first kind [Burmin, 1993, 1996]

$$\int_q^{q_0}\frac{x(p,z^*)dp}{\sqrt{w^2-p^2}} = \int_q^{q_0}\lambda(u)K_1(u,w)du + \int_{\bar{u}^*}^{u^*}K_2(u,w)dH(u),$$
$$\int_q^{q_0}\frac{t(p,z^*)pdp}{\sqrt{w^2-p^2}} \quad (7)$$
$$= \int_q^{q_0}\lambda(u)u^2K_1(u,w)du + \int_{\bar{u}^*}^{u^*}u^2K_2(u,w)dH(u)$$

under the conditions

$$\lambda(u) = -dz/du \ge 0, \quad dH(u) \ge 0, \quad (8)$$

where

$$K_1(u,w) = 2\ln\frac{\sqrt{w^2-q^2}+\sqrt{u^2-q^2}}{\sqrt{w^2-u^2}},$$
$$q \le u \le q_0 \le \bar{u}^* \le w \le u^*,$$
$$K_2(u,w) = 2\ln\frac{\sqrt{w^2-q^2}+\sqrt{u^2-q^2}}{\sqrt{w^2-q_0^2}+\sqrt{u^2-q_0^2}},$$
$$q \le q_0 \le u^* \le u, \; w \le u^*.$$

Here, $\lambda(u)$ determines the velocity distribution below a waveguide, and the function $H(u)$ is defined in the waveguide. The problem (7)–(8) reduces to a problem of quadratic programming and is solved by numerical methods, in particular, by the method of conjugate gradients.

As mentioned above, the inverse kinematic problem does not have a unique solution in a medium with waveguides and is therefore a classic ill-posed problem. However, if the velocity functions are monotonic, the refraction traveltime curve provides a unique solution to problem (7)–(8) [Burmin, 1996].

An important property of problem (7)–(8) is the fact that the functions $\lambda(u)$ and $H(u)$ can be reconstructed from any fragment of the traveltime curve.

In some cases, taking into account the approximate nature of the initial data, problem (7)–(8) can be substantially simplified if the velocity in a waveguide is assumed to be constant or vary by a linear law. Let z^* and $\bar{z}^*$ be the upper and lower boundaries of a waveguide. Then, if $v(z) = v^* = \text{const}$ at $z \in [z^*, \bar{z}^*]$, i.e., the velocity in the waveguide is constant, the waveguide parameters (the velocity in the waveguide v^* and its thickness Δz^*) are determined by the formulas [Burmin, 1993]

$$v^* = \sqrt{v(\bar{z}^*)\bar{v}},$$
$$\Delta z^* = \frac{\Delta x}{2}\sqrt{\frac{v(\bar{z}^*)}{\bar{v}}-1}, \quad \bar{z}^* = z^* + \Delta z^*, \quad (9)$$

where $\bar{v} = \Delta x/\Delta t$ and $v(\bar{z}^*)$ are determined from the traveltime curve. Note that constant-velocity waveguides whose parameters satisfy the relations for Δx and Δt are most widespread.

A system of two nonlinear equations describes a linear variation law in a waveguide:

$$\Delta x = \frac{2\Delta z^*}{v(z^*) - v^*}\sqrt{v^2(z^*) - v^{*2}},$$
$$\Delta t = \frac{2\Delta z^*}{v(z^*) - v^*}\ln\frac{v(z^*) + \sqrt{v^2(z^*) - v^{*2}}}{v^*}. \quad (10)$$

Solving this system, we obtain the sought-for values v^* and Δz^*. In this case, Δx, Δt, and $v(z^*)$ are also determined from traveltime data. A linear function approximates here the monotonic velocity function in the waveguide.

In transforming the velocity function on the half-plane into the velocity function on the circle, the constant velocity v_0 is transformed into a linear function increasing with radius (decreasing with depth),

$$v(r) = v_0\frac{r}{R}, \quad (11)$$

and the linear function is transformed into the function $v(z) = v_0 + g(z - z_0)$

$$v(r) = r\left(\frac{v_0}{R} + g\ln\frac{r_0}{r}\right), \quad (12)$$

where v_0 is the wave velocity at the distance r_0 from the Earth's center and g is a constant determining the velocity gradient on the plane. With $g > 0$, function (12) can either increase or decrease with depth, depending on the value of g.

3. Inversion of a Reflection Traveltime Curve

The inversion of a reflection traveltime curve in the case of a vertically heterogeneous medium reduces to either the minimization of the functional [Burmin, 1992, 1993]

$$J(H, x) = \int_{p_1}^{p_2}\frac{1}{p}\left\{x(p) - p\int_{p_2}^{u_{max}}\frac{dH(u)}{\sqrt{u^2 - p^2}}\right\}^2 dp \quad (13)$$

under the condition

$$dH(u) \geq 0, \quad (14)$$

where $u(z) = v^{-1}(z)$, or the solution of the Fredholm integral equation of the first kind with a positive symmetric kernel with respect to the function $H(u)$:

$$f(w) = \int_{p_2}^{u_{max}}\ln\frac{\sqrt{w^2 - p_1^2} + \sqrt{u^2 - p_1^2}}{\sqrt{w^2 - p_2^2} + \sqrt{u^2 - p_2^2}}dH(u),$$

where $f(w) = x_{max}\arcsin\frac{p_2}{w} - x_{min}\arcsin\frac{p_1}{w} - \int_{x_{min}}^{x_{max}}\arcsin\frac{t'(x)}{w}dx$; $0 \leq p \leq u^* \leq u$, $w \leq u_0$.

The values $u_{min} = p_2$ and u_{max} can be specified arbitrarily in the intervals $[0, u(z^*)]$ and $[u(0), \infty]$, and x_{min} and x_{max} are the values at the end points of the traveltime curve. The minimization of functional (13) under linear restraints (14) also reduces to a problem of quadratic programming.

The inversion problem for a refraction traveltime curve in a medium with a waveguide was shown to have a unique solution in the class of monotonic velocity functions. In the reflection case, the solution is also unique in the class of monotonic functions. Moreover, similarly to the refraction inversion, the velocity curve above a reflector can be reconstructed from any fragment of the reflection traveltime curve. In the general case, the minimum velocity value and the thickness of the layer above the reflector are determined uniquely.

4. Smoothing of Observed Traveltime Data by Convex Splines for Seismic Waves Traveling in a Vertically Heterogeneous Medium

Notwithstanding a fairly well-developed theoretical background, practical application of the formulas obtained for the inversion of traveltime data encounters a difficulty consisting in that these formulas are inapplicable to actually observed traveltime data, because the latter never meet the solvability conditions of the problem and the kinematic inversion of seismic data is an essentially ill-posed problem [Burmin, 1993].

The use of the formulas for the inversion of seismic traveltime data implies that the ray parameter (the derivative of the traveltime curve) is specified for each value $x \in [0, x_M]$. Only the values x and t are assumed to be known from observations, and this raises the problem of differentiation of an experimental traveltime curve represented by a discrete set of points with uncertainties. Generally, this problem is also ill-posed.

In order to solve the problem using the inversion formulas for a traveltime curve, an experimental traveltime curve should be preliminarily smoothed by a function $T(x)$ meeting conditions imposed on traveltime curves and having a minimum deviation from the experimental data in a given metric $\rho = (f, \tilde{f})$. Differentiating the resulting smoothing function, we can find the values of the parameter p.

Note that the experimental traveltime curve is specified as a discrete set of points given on an observation network Δ: $a = x_0, \ldots, x_n = b$. However, the application of the inversion formulas requires that the function $t(x)$ be specified on the entire interval $[a, b]$. Therefore, the experimental traveltime curve should be complemented in the intervals $[x_j, x_{j+1}]$. Evidently, the smoothing function should solve this problem as well.

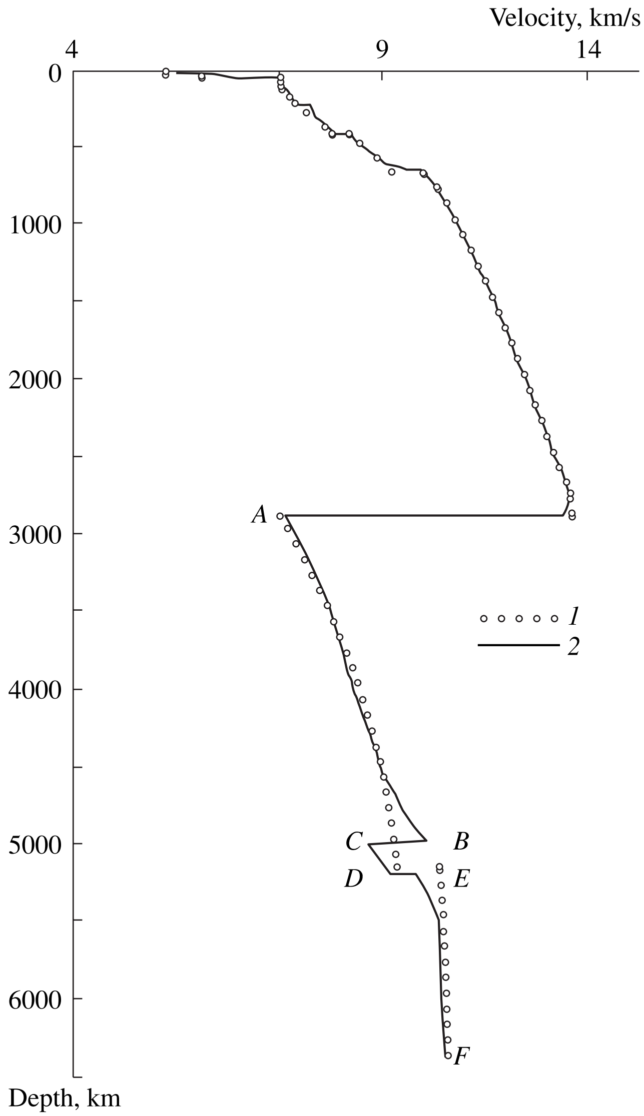

Fig. 6. Distribution of the *P* wave velocity in the Earth: (*1*) IASPEI91 model; (*2*) model with the function $v(r)/r$ monotonically decreasing in the radius interval 2652–3478 km (depths of 3719–2893 km).

As mentioned above, in intervals where the function $t(x)$ is monotonic and continuous, it should meet the following requirements that are necessary for $t(x)$ to be a traveltime curve of refracted or reflected waves.

(i) $t(x)$ is a nonnegative function, $t(x) \geq 0$ ($x \in [0, x_M]$).

(ii) The derivative of $t(x)$ is a nonnegative function, $t'(x) \geq 0$.

(iii) If the function $t(x)$ in an interval $[x_j, x_{j+1}]$ describes a normal branch of the traveltime curve, the second derivative $t''(x)$ is a nonnegative function, $t''(x) \leq 0$. If the function $t(x)$ in an interval $[x_j, x_{j+1}]$ describes a reversed branch of the traveltime curve, the second derivative $t''(x)$ is a positive function, $t''(x) > 0$. In the case of traveltime curves of reflections and waves traveling upward from the source, the second derivative of $t(x)$ must be a nonnegative function, $t'' \geq 0$ [Burmin, 1980a].

Evidently, the first requirement is not necessarily met. The second requirement is met almost always, except for the cases when the determination uncertainties of the traveltime curve are inadmissibly large. The third requirement is actually never met, because even small errors in determinations of times and distances result in the violation of the conditions $t''(x) \leq 0$ ($t''(x) > 0$) for refractions or $t''(x) \geq 0$ for reflections. Moreover, the lateral heterogeneity of the real elastic medium also results in the violation of the third requirement.

The conditions $t''(x) \leq 0$ ($t''(x) \geq 0$) and $t''(x) > 0$, which should be met by refraction (reflection) traveltime curves, imply that $t(x)$ are functions convex upward or downward, and therefore, the approximating functions T(x) should take into account the character and degree of smoothness of $t(x)$ in a sufficiently large domain of definition of $t(x)$.

The functions most suitable for approximating the experimental traveltime data are convex cubic splines, because they possess distinct local properties: solely the values t_i defined at points close to x_i have a significant effect on the behavior of the function $T(x)$. Lower order splines fail to ensure adequate accuracy, whereas higher order splines violate the convexity condition [Burmin, 1980a].

Our problem will be solved if we succeed in constructing a spline minimizing the functional

$$S = \sum_{i=1}^{N-1} \left\{ \sum_{j=1}^{N-1} \alpha_{ij} T''_j - \tilde{t}_i \right\}^2 \tag{15}$$

under the convexity condition

$$T''_i \leq 0 \quad (T''_i \geq 0). \tag{16}$$

Problem (15), (16) also reduces to a problem of quadratic programming.

It is noteworthy that all of the three problems considered above reduce to problems of quadratic programming. As is well known, a problem of quadratic programming always has a unique solution because, for example, the functional S convex in T is bounded from below and continuous on the convex set $T = \{T'' \mid T'' \leq 0$ $(T'' \leq 0)\}$ [Karmanov, 1975].

ANALYSIS OF THE IASPEI91 VELOCITY MODEL

Figure 2 presents the theoretical traveltime curves calculated from the IASPEI91 model (Fig. 6) and superimposed on the record section. Here, the traveltime branch AB of the refracted wave traveling in the outer core and denoted usually as $PKP2$ is interpreted as a reversed branch in which A is the initial point and B is the critical point. We should note that the initial point lies below and to the left of the visible vibrations and the critical point is above and to the right of the final arrivals of this wave.

The first circumstance is related to the fact that, according to the model, the position of the initial point is determined not only by the velocity distribution in the outer core but also by the velocity value in the lower mantle, namely, at its base in the *D*'' layer. Actually, the penetration depth of the wave traveling in the outer core is subjected to the condition that the ratio $v(r)/r$ at this depth has the same value as in the overlying layer. This implies that the velocity, equal to 13.691 km/s at the base of the *D*'' layer in the IASPEI91 model, is evidently overestimated. In the PREM model, the *P* wave velocity at this depth is 13.717 km/s.

The second circumstance indicates that the actual velocity in the lower part of the outer core is higher than its value accepted in the IASPEI91 model.

The traveltime points *B* and *C* are connected by the branch *CB*. However, as seen from Fig. 2, this interval does not contain visible vibrations that could be regarded with confidence as regular vibrations. This fact was well appreciated by Jeffreys and induced him to introduce a lower velocity layer in the lowermost outer core.

The branch *CD* is the traveltime curve of the reflection from the outer/inner core boundary. The curve is seen to lie below the related vibrations, indicating that either the model boundary depth is smaller than its real value or the velocity value in the overlying layer is overestimated.

The branch *EF*, usually referred to as *PKP*1, is the traveltime curve of the wave refracted in the inner core. Here, it also lies below the visible vibrations and, in addition, its critical point is located at a distance of about 177° rather than 180°, as should be expected.

Thus, the IASPEI91 model traveltime curves are not consistent everywhere with the visible vibrations in the record section. This is due in part to the fact that the IASPEI91 model incorrectly interprets seismic waves and, in particular, precursors.

CORRELATION AND INTERPRETATION OF WAVES

As seen from the record sections, presented in Figs. 4 and 5, vibrations of the first event crossing the outer core appear clearly, albeit with a small relative amplitude, at an epicentral distance of 176° and are gradually transformed into precursors at 142°, continuing until an epicentral distance of about 134°. The point *A* is the initial point of the traveltime branch *AB*. Since the point *B* is the critical point of the branch, the traveltime curve evidently has a discontinuity due to the presence of a waveguide (the F zone) immediately above the inner core (the G zone).

Vibrations related to the traveltime branch *CD* are due to the refraction traveling in the lower part of the F zone, the reflection from the inner core, or both (the reflection branch is continuously transformed into the refraction branch).

The branch *EF* is an envelope of waves traveling in the inner core.

Although the quality of worldwide network records is not very good (some traces do not fit the general pattern and their time errors reach 1 s), their accuracy is sufficient, without going into detail, for the interpretation of the wavefield at given epicentral distances.

Based on the above correlation of seismic waves, we constructed experimental traveltime curves of refractions (branches *AB* and *EF*) and reflections (*CD*) (see Fig. 4). These curves were used for solving the inverse kinematic problem. Note that Fig. 4 presents theoretical traveltime curves that are in full agreement with the experimental data.

RADIAL DISTRIBUTION OF VELOCITY IN THE CORE

To determine velocities in the core, the distributions of body waves in the crust and mantle were taken from [Burmin *et al.*, 1992; Burmin, 1994]. The inverse problem was solved using the traveltime data of *P* waves traveling in a medium that includes a lower velocity layer. The method described above and implemented in a FORTRAN-77 program was applied for this purpose.

The velocity in the upper part of the outer core was determined with the use of a simplified, more robust scheme. The velocity curve, determined in the class of linear functions approximating an arbitrary monotonic velocity function, was then converted into a function of type (12).

To determine the velocity curve more accurately, we solved the forward kinematic problem for a spherically symmetric Earth; i.e., a given velocity cross section was used for constructing the traveltime curve to be fitted to experimental data. Results of this solution showed that the velocity cross section, obtained under the assumption that the velocity variation in the waveguide is governed by law (12), can be regarded as a solution to the inverse problem.

Here, we discuss in more detail the resulting velocity curve (Fig. 6). First of all, note that the velocity at the upper boundary of the outer core amounts to 8.1 km/s. This value is greater than in the IASPEI91 model (8.0 km/s) and close to the PREM value (8.105 km/s).

The shadow zone in the outer core extends to a depth of 3956 km. At this depth, the seismic velocity gradient starts to increase as compared with the overlying layer. The initial point of the corresponding traveltime branch *AB* (Fig. 4) is located at an epicentral distance slightly smaller than 180°.

We should emphasize that the maximum penetration depth of the first ray arriving at the Earth's surface, as well as the position of the initial traveltime point corre-

Distribution of the P wave velocity in the Earth's core

H, km	v_P, km/s	H, km	v_P, km/s	H, km	v_P, km/s	H, km	v_P, km/s
2893.000	8.100	3462.378	8.950	4014.650	9.430	4619.507	10.130
2913.000	8.137	3479.104	8.968	4034.650	9.457	4639.062	10.167
2932.885	8.173	3512.268	9.003	4054.480	9.484	4658.400	10.182
2952.656	8.209	3528.707	9.020	4074.142	9.519	4677.522	10.225
2972.313	8.243	3545.051	9.036	4093.637	9.533	4696.430	10.256
2991.856	8.277	3561.301	9.052	4112.967	9.556	4715.127	10.285
3011.288	8.311	3577.459	9.068	4132.132	9.578	4733.615	10.312
3030.608	8.344	3593.523	9.083	4151.135	9.599	4751.897	10.337
3049.817	8.376	3609.494	9.097	4169.977	9.619	4769.975	10.360
3068.915	8.407	3625.374	9.111	4188.659	9.637	4777.960	10.370
3087.903	8.438	3641.163	9.124	4207.182	9.655	4797.960	10.412
3106.782	8.469	3656.861	9.137	4225.547	9.672	4817.709	10.451
3125.553	8.498	3672.468	9.150	4243.757	9.688	4837.210	10.488
3144.216	8.527	3687.986	9.162	4278.540	9.727	4856.466	10.522
3162.771	8.556	3703.414	9.174	4298.351	9.762	4867.170	10.540
3181.220	8.583	3718.754	9.186	4317.974	9.777	4887.170	10.592
3199.562	8.611	3734.006	9.198	4337.411	9.799	4906.904	10.641
3217.800	8.637	3749.170	9.210	4356.664	9.821	4923.640	10.680
3235.932	8.663	3764.246	9.222	4375.735	9.841	4943.640	10.743
3253.960	8.689	3779.236	9.234	4394.626	9.860	4963.364	10.802
3271.884	8.713	3794.140	9.246	4413.337	9.878	4983.640	10.860
3289.706	8.738	3808.958	9.258	4431.872	9.894	5197.321	10.150
3307.424	8.761	3823.691	9.270	4450.231	9.910	5197.321	10.640
3325.041	8.784	3838.339	9.282	4468.416	9.924	5357.33	10.91
3342.557	8.807	3852.903	9.294	4486.429	9.937	5393.25	10.96
3359.972	8.829	3867.384	9.306	4504.271	9.959	5456.21	11.03
3377.286	8.850	3881.780	9.318	4522.660	9.970	5505.68	11.08
3394.502	8.871	3896.094	9.330	4542.660	9.993	5925.56	11.12
3411.618	8.892	3910.326	9.342	4562.444	10.024	6371.028	11.20
3428.636	8.912	3924.476	9.354	4579.730	10.050		
3445.556	8.931	3938.545	9.384	4599.730	10.091		

sponding to this ray, is essentially dependent on the velocity values directly above the waveguide. The PREM and IASPEI91 velocity values in the lower part of the D'' layer amount to ≈13.7 km/s. This value determines both the maximum penetration depth of the first ray (3971 km) and the position of the initial point of the corresponding traveltime branch (162.3°). The mantle/core boundary depth and the velocity at the lower boundary of the mantle were obtained in [Burmin, 1994] by the inversion of PcP traveltime data (≈2893.0 km and ≈13.54 km/s, respectively). The updating of the velocity curve with the use of the solution of the forward kinematic problem showed that a value of 13.50 km/s fits best the velocity at the lower boundary of the mantle.

Starting with a depth of 4600 km, the velocity gradient further increases, yielding a significant curvature of the traveltime curve in the interval of epicentral distances 145°–134°. In this interval, the traveltime curve envelops very accurately the first arrivals of precursors.

In the depth interval 4983.64–5000.0 km (BC), the P wave velocity sharply drops from 10.86 to 9.7 km/s. This velocity drop accounts for the gap in the traveltime curve in the interval of epicentral distances 134°–154°. Possibly, a small velocity jump takes place here. In this case, the traveltime branch AB is continuously transformed into a reflection traveltime curve (not shown in Figs. 4 and 5) extending toward smaller epicentral distances.

The traveltime branch of the wave traveling in the layer CD starts at a distance of 154°. The refraction traveltime curve is continuously transformed at 152.2° into the traveltime curve (the interval CD (Fig. 4)) of the reflection from the CD layer base (Fig. 6). The velocity at the boundary DE (a depth of 5197.3 km) changes from 10.15 to 10.64 km/s. The depth of the boundary DE and the velocity in the layer CD were determined by the inversion of reflection traveltime data using the method described above. The velocity in this layer increases with depth, and the velocity gradient is fitted in such a way that the critical point of the reflection traveltime curve is located at an epicentral distance of ≈152°. Formally, the velocity could decrease, similar to the Jeffreys model, but in this case the critical reflection point would shift to epicentral distances as large as 180°, which is at variance with the observed data.

Below the boundary DE, the velocity first increases rather strongly to a depth of about 5500 km, after which its increase toward the Earth's center becomes nearly linear, with a gradient slightly larger than in the standard model (Fig. 6). The velocity at the Earth's center is 11.2 km/s. Its values in the PREM and IASPEI91 models are 11.6 and 11.4, respectively.

The results of the P wave velocity determination in the core are presented in the table.

CONCLUSION

Using forward methods for the kinematic inversion of seismic data, the interpretation of record sections of seismic waves from deep earthquakes recorded by the worldwide network provided the P wave velocity distribution in the core best fitting the observed wave patterns. The commonly used optimization methods of the inversion, involving a linearization of the problem, fail to provide such results because they require a sufficiently good initial approximation. The situation is no better with trial-and-error methods, which require large experience and intuition of an experimenter.

As mentioned above, an infinite set of velocity curves consistent with observed data can be constructed for a waveguide. However, only one of these curves is a monotonically increasing curve. We constructed such a curve in the outer core (interval AB in Fig. 6), and most likely, it is physically meaningful. However, nonmonotonic behavior of the velocity function is also possible in some intervals. We should note that the set of permissible velocity curves is not a continuous band, as it is sometimes displayed, but a more complex geometric figure. An example illustrating the construction of the set of linear functions consistent with observed traveltime data is given in [Burmin, 1978].

A characteristic feature of the velocity curve in the interval AB is a large velocity gradient in the 500-km interval ending at the point B (Fig. 6).

The presence of the lower velocity layer CD is absolutely indispensable for bringing theoretical traveltime curves into agreement with observed data. The sharp velocity decrease in this layer is possibly evidence for a variation in the material composition increasing the density without a change in the aggregate state.

Below the boundary DE, material is transformed into a solid state, giving rise to an abrupt rise in the P wave velocity. The P wave velocity distribution in the inner core is fairly simple, particularly, near the Earth's center, but this might be due to deficient data here. Additional data at epicentral distances greater than 172° can provide more reliable constraints on the velocity distribution in the inner core.

The use of short-period records for the velocity determination in the core enabled the interpretation of precursors well consistent with observed data. Evidently, the 500- and 200-km layers in the lower part of the outer core have no signatures in short-period records. For example, the 30-s waves have a wavelength of about 300 km here, which is comparable with the thicknesses of these layers. Therefore, precursors are only recognizable in short-period records. Even only because their intensity is higher than the intensity of waves in the initial part of the branch EF (Fig. 3), precursors cannot be scattered waves.

In addition to the subject of this paper, the distribution of S waves in the inner core is also important. The pertinent available data are virtually absent, and the elucidation of this problem requires a large volume of work. In this respect, the following statement of Puzyrev [2000, p. 1489] is appropriate: "… a detailed study of the inner core using P and S waves is a task of prime importance, and its accomplishment is indispensable for success in the study of the origin and evolution of the Earth."

ACKNOWLEDGMENTS

I am grateful to A.V. Timofeeva, a student of the Physical Faculty of Moscow State University, for her help at the preliminary stage of the preparation of this paper, including the review of publications on the transition F zone. I also thank N.A. Sergeeva, who kindly afforded me data of the worldwide seismological network, and Yu.V. Tulina for helpful discussion.

REFERENCES

1. R. D. Adams and M. J. Randall, "Observed Triplication of *PKP,"* Nature (London) **200**, 744–745 (1963).
2. R. D. Adams and M. J. Randall, "The Fine Structure of the Earth's Core," Bull. Seismol. Soc. Am. **54** (5), 129–131 (1964).
3. L. M. Antonova, "Parametrization of the Seismic Section in the Inner–Outer Core Transition Zone," in *Computational Seismology* (1971), Vol. 5, pp. 260–271 [in Russian].
4. I. Ya. Azbel' and T. B. Yanovskaya, "Determination of Possible Velocity Sections in the Transition Zone of the

Core," in *Computational Seismology* (1975), Vol. 4, pp. 199–217 [in Russian].

5. B. A. Bolt, "Gutenberg's Early *PKP* Observations*,"* Nature (London) **196**, 121–124 (1962).
6. B. A. Bolt, "The Velocity of Seismic Waves Near the Earth's Center," Bull. Seismol. Soc. Am. **54** (1), 191–208 (1964).
7. G. G. G. Buchbinder, "A Velocity Structure of the Earth's Core," Bull. Seismol. Soc. Am. **61** (2), 429–456 (1971).
8. K. E. Bullen, *The Earth's Density* (Wiley, New York, 1975; Mir, Moscow, 1978).
9. V. Yu. Burmin, "Determination of the Seismic Wave Velocity in a Waveguide from Refraction Traveltime Data," Izv. Akad. Nauk SSSR, Fiz. Zemli, No. 7, 86–89 (1978).
10. V. Yu. Burmin, "Approximation of Seismic Traveltime Data by Convex Splines," Izv. Akad. Nauk SSSR, Fiz. Zemli, No. 2, 90–96 (1980a).
11. V. Yu. Burmin, "Inversion Formulas for Discontinuous Refraction Traveltime Curves," Izv. Akad. Nauk SSSR, Fiz. Zemli, No. 6, 94–100 (1980b).
12. V. Yu. Burmin, "Numerical Inversion of Reflection Traveltime Data," Geofiz. Zh. **14** (1), 72–81 (1992).
13. V. Yu. Burmin, *Methods for Numerical Inversion of Seismic Traveltime Data* (Nauka, Moscow, 1993) [in Russian].
14. V. Yu. Burmin, "Determination of the Seismic Wave Velocity in the Mantle and the Earth's Core Radius from *P* and *PcP* Traveltimes," Vulkanol. Seismol., No. 4, 84–97 (1994).
15. V. Yu. Burmin, "Inversion of a Discontinuous Refraction Traveltime Curve," Fiz. Zemli, No. 10, 59–64 (1996).
16. V. Yu. Burmin, L. A. Savrina, and Yu. V. Kugaenko, "Velocity Section of the Upper Mantle in the Sea of Okhotsk Region from Deep Earthquake Data," Vulkanol. Seismol, No. 2, 64–75 (1992).
17. J. R. Cleary and R. A. W. Haddon, "Seismic Wave Scattering Near the Core–Mantle Boundary: A New Interpretation of Precursors to *PKIKP,"* Nature (London) **240**, 549–551 (1972).
18. D. J. Doornbos and N. J. Vlaar, "Regions of Seismic Wave Scattering in the Earth's Mantle and Precursors to *PKP*," Nature (London) **243**, 58 (1973).
19. A. M. Dziewonski and D. Anderson, "Preliminary Reference Earth Model," Phys. Earth Planet. Inter. **25**, 297–356 (1981).
20. K. Ergin, "Seismic Evidence for a New Layered Structure of the Earth's Core," J. Geophys. Res. **72** (14), 3669–3688 (1967).
21. V. S. Geiko, "Theory of the Refraction Traveltime Data Inversion," Geofiz. Zh. **4** (2–3), 3–17 (1982).
22. M. L. Gerver and V. M. Markushevich, "Seismic Wave Velocity Determination from Traveltime Data," in *Computational Seismology* (Nauka, Moscow, 1967), Vol. 3, pp. 3–51 [in Russian].
23. B. Gutenberg, *Physics of the Earth's Interior* (New York, 1959; IL, Moscow, 1963).
24. B. Gutenberg, "The "Boundary of the Earth's Inner Core," Trans. Amer. Geophys. Un. **38**, 750–753 (1957).
25. B. Gutenberg, "Wave Velocities in the Earth's Core," Bull. Seismol. Soc. Am. **48**, 301–314 (1958a).
26. B. Gutenberg, "Caustics Produced by Waves through the Earth's Core," J. Geophys. **1**, 238–248 (1958b).
27. B. Gutenberg and C. F. Richter, "*P* and the Earth's Core," Month. Not. R. Astron. Soc. (Geophys. Suppl.) **4** (5), 363–372 (1938).
28. B. Gutenberg and C. F. Richter, "On Seismic Waves," Gerl. Beitr. Geophys. **54**, 94–136 (1939).
29. N. Hai, "Propagation des Ondes Longitudinales dans le Noyeau Terrestre," Ann. Geophys. **19**, 258–346 (1963).
30. A. L. Hales and J. L. Roberts, "The Velocities in the Outer Core," Bull. Seismol. Soc. Am. **61** (4), 1051–1059 (1971).
31. M. A. H. Hedlin and P. M. Shearer, "Probing Mid-Mantle Heterogeneity Using *PKP* Coda Waves," Phys. Earth Planet. Inter. **130**, 195–208 (2002).
32. J. A. Jacobs, *The Earth's Core* (Wiley, New York, 1976; Mir, Moscow, 1979).
33. H. Jeffreys, "The Times of the Core Waves," Month. Not. R. Astron. Soc. (Geophys. Suppl.) **4** (7), 548–561 (1939a).
34. H. Jeffreys, "The Times of the Core Waves (Second Paper)," Month. Not. R. Astron. Soc. (Geophys. Suppl.) **4** (7), 594–615 (1939b).
35. H. Jeffreys, *The Earth. Its Origin, History, and Physical Constitution* (Cambridge Univ. Press, Cambridge, 1959; IL, Moscow, 1960).
36. V. G. Karmanov, *Mathematical Programming* (Nauka, Moscow, 1975) [in Russian].
37. B. L. N. Kennett, *IASPEI 1991. Seismological Tables* (Res. School Earth Sci., Australian Nat. Univ., 1992).
38. A. Morelli, A. M. Dziewonski, and J. H. Woodhouse, "Seismic Evidence for an Inner Core Transition Zone," Geophys. Res. Lett. **13**, 154 (1986).
39. R. D. Oldham, Quart. J. Geol. Soc., **62** (1906).
40. N. N. Puzyrev, "On Some Problems of General Seismology," Geol. Geofiz. **41** (11), 1487–1491 (2000).
41. A. Qamar, "Revised Velocities in the Earth's Core," Bull. Seismol. Soc. Am. **63** (3), 1073–1105 (1973).
42. M. J. Randall, "*SKS and Seismic Velocities in the Outer Core,*" Geophys. J. R. Astron. Soc. **21** (5), 441–445 (1970).
43. E. F. Savarenskii and D. P. Kirnos, *Fundamentals of Seismology and Seismometry* (Gostekhizdat, Moscow, 1955) [in Russian].
44. X. Song and D. V. Helmberger, "Seismic Evidence for an Inner Core Transition Zone," Science **282**, 924–927 (1998).
45. L. Stixrude and J. M. Brown, "The Earth's Core," Rev. Miner., 261–282 (1999).
46. K. Wadati and K. Masuda "On the Travel Time of Earthquake Waves," Geophys. Mag. (Tokyo) **8**, 187–194 (1934).
47. T. B. Yanovskaya, "Some Data on the Seismic Waves in the Earth's Core," in *Computational Seismology* (1975), Vol. 5, pp. 272–281 [in Russian].