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P Wave Velocities and Layer Thicknesses Determined from the Difference between Traveltime Curves of Converted *S* and Direct *P* Waves

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Abstract—Two Fredholm first-kind integral equations relating the *P* wave velocity to the travel time difference between converted *SV* and *P* waves are derived for the inversion of difference between the traveltime curves of converted *SV* and *P* waves propagating in an elastic medium, i.e., for the determination of the *P* velocity and localization of the refracting interface. Numerical modeling examples of determining the velocity distribution in a layer and localizing the refracting interface are presented.

INTRODUCTION

The refraction method is most commonly used to determine the variation of seismic wave velocities in an elastic medium. A limitation of this method is that the first arrival curves are typically obtained for shallow wave penetration depths, particularly if velocity gradients in layers are small. For this reason, the first arrival curves are used to determine the seismic velocities only in the uppermost layers. More complete information about the velocity distribution can be recovered from the interpretation of reflection traveltime curves. In this case, the velocity distribution can be found down to the reflecting interface. However, using the reflection and refraction methods to study the velocity structure of the Earth is rather expensive, in particular, because it requires large explosions to be fired, which are therefore limited in number.

An alternative to these methods is the method of converted waves from earthquakes (MCWE) [Andreev, 1957; Bulin, 1960; Pomerantseva and Mozzhenko, 1977; Gal'perin *et al.*, 1995], which has been widely applied in seismological practice to determine the Earth's structure. Nevertheless, a rigorous theory of inversion of travel time differences between *P* and converted *S* waves has not been developed until recently. Gal'perin *et al.* [1995] directly stated that "it is impossible to determine elastic wave velocities from MCWE data." Usually, these data, combined with average *P* and *S* velocities derived from other observations are only used to estimate the refractor depth interface. For this purpose, the Hasegawa [1930] formula (see also [Pomerantseva and Mozzhenko, 1977; Gal'perin *et al.*, 1995]) or other relatively simple formulas [Andreev, 1957; Bulin, 1960] are widely used. Based on the Hasegawa formula, Akhmerov [1974] proposed to determine both the average *P*-wave velocity in a layer and the refractor location from observations at a single point. In his approach, the refracting interface is localized by making use of *P* and converted *S* waves recorded at a single point, but reflected from different points on the refracting surface (i.e., the related *P* waves traveled along different paths to the surface. However, in this case, the distance between the refraction points remains unknown, which gives rise to determination uncertainties in the velocities above the refractor, and the formula itself is approximate.

In this paper, we propose to determine the velocity in a layer and its thickness by using the points on traveltime curves of *P* and converted *S* waves from a single refraction point that have the same ray parameter, i.e., corresponding to the same ray arriving to the refraction point. To solve thus formulated problem we assume that the ratio between *P*- and *S*-wave velocities above the refractor is constant and known and derive the Fredholm first-kind integral equations relating the *P* wave velocity and the difference between traveltime curves of *SV* and *P* waves propagating in a vertically heterogeneous elastic medium.

INTEGRAL EQUATIONS FOR DETERMINING THE P-WAVE VELOCITY IN A VERTICALLY HETEROGENEOUS MEDIUM

Let a source of elastic oscillations be placed at a point within an elastic medium. We consider the situation in which a *P* wave striking a layer boundary from below produces a converted *SV* wave, and both *SV* and *P* waves are recorded on the surface. The following assumptions are adopted: (1) the Earth's surface is plane in the region of refraction and reception of the waves, (2) the seismic wave velocity is a piecewise twice differentiable depth-dependent function $v = v(z)$ bounded in a finite depth interval $[0, z]$, and (3) the ratio of P to SV wave velocities is known and equal to $k = v_P/v_S$.

The equations describing the differences between epicentral distances and between travel times of SV and P waves traveling from the refractor $z = z^*$ to the receiver point can be written in the parametric form

$$x_P(p) - x_S(p) = \Delta x(p) = \int_0^{z^*} \left\{ \frac{p}{\sqrt{u_P^2(z) - p^2}} - \frac{p}{\sqrt{u_S^2(z) - p^2}} \right\} dz, \quad (1)$$

$$t_S(p) - t_P(p) = \Delta t(p) = \int_0^{z^*} \left\{ \frac{u_S^2(z)}{\sqrt{u_S^2(z) - p^2}} - \frac{u_P^2(z)}{\sqrt{u_P^2(z) - p^2}} \right\} dz, \quad (2)$$

where $u_P(z) = v_P^{-1}(z)$ and $u_S(z) = v_S^{-1}(z)$ are the slowness values of P and SV waves, respectively (by analogy with optics, we call these values the refraction indexes with respect to the unit velocity; and p is the P and SV ray parameter, $0 \le p \le u_P(z^* + 0)$. It is assumed that $x_P(p)$, $x_S(p)$ and $t_P(p)$, $t_S(p)$ correspond to the same point on the refracting surface, but to different reception points of the direct P and converted SV waves.

In the problem considered, the direct P and converted SV waves traverse the entire layer $0z^*$; i.e., seismic rays have no turning points in this layer. This means that the travel times and the refraction point–receiver distances do not change if the layer $0z^*$ is divided into elementary layers and if the latter are interchanged. Consequently, $v_P(z)$ and, accordingly, $v_S(z)$ cannot be uniquely determined from the travel time difference between converted SV and direct P waves; in other words, the same travel rimes of P waves $t(p)$ correspond to different velocity functions $v_P(z)$ having the same measure $H(u)$ [Gerver and Markushevich, 1967]

$$H(u) = \mathrm{mes}\{z\colon z \le z^*,\ v_P^{-1}(z) \le u\},$$

By definition, the function $H(u)$ (a) does not decrease, (b) is zero for $-\infty < u \le u^* = u(z^* - 0)$, and (c) is $h = z^*$ for $u(0) = u_0 \le u < \infty$. Here, u^* and u_0 are the minimum and maximum values, respectively, of the function $u(z) = v_P^{-1}(z)$ in the layer above the refractor.

Now, we find the function $H(u)$ from equations (1) and (2), and thereby the refractor depth, as well as the minimum and maximum velocities of the waves above the interface.

Setting $u(z) = u_P(z)$, (1) and (2) take the form

$$\Delta x(p) = \int_0^{z^*} \left\{ \frac{p}{\sqrt{u^2(z) - p^2}} - \frac{p}{\sqrt{k^2u^2(z) - p^2}} \right\} dz, \quad (3)$$

$$\Delta t(p) = \int_0^{z^*} \left\{ \frac{k^2u^2(z)}{\sqrt{k^2u^2(z) - p^2}} - \frac{u^2(z)}{\sqrt{u^2(z) - p^2}} \right\} dz. \quad (4)$$

We write (3) and (4) in terms of the Stieltjes integrals

$$\Delta x(p) = p\int_{u^*}^{u_0} \left\{ \frac{1}{\sqrt{u^2 - p^2}} - \frac{1}{\sqrt{k^2u^2 - p^2}} \right\} dH(u), \quad (5)$$

$$\Delta t(p) = \int_{u^*}^{u_0} \left\{ \frac{k^2u^2}{\sqrt{k^2u^2 - p^2}} - \frac{u^2}{\sqrt{u^2 - p^2}} \right\} dH(u), \quad (6)$$

where $0 \le p \le u(z^* + 0) = u^* \le u \le u_0$; $dH(u) \ge 0$.

Multiplying the right- and left-hand sides of (6) by $p/\sqrt{w^2 - p^2}$ and integrating over p from p_1 to p_2, we find

$$g(w) = \int_{p_1}^{p_2} \frac{\Delta t(p)pdp}{\sqrt{w^2 - p^2}} = \int_{p_1}^{p_2} \frac{p}{\sqrt{w^2 - p^2}} \times \left\{ \int_{u^*}^{u_0} \left\{ \frac{k^2u^2}{\sqrt{k^2u^2 - p^2}} - \frac{u^2}{\sqrt{u^2 - p^2}} \right\} dH(u) \right\} dp,$$

where $0 \le p_1 \le p \le p_2 \le u^* \le w \le u_0$.

The integrals in the right-hand side of this equation can be interchanged, which yields

$$g(w) = k^2 \int_{u^*}^{u_0} u^2 dH(u) \int_{p_1}^{p_2} \frac{pdp}{\sqrt{(w^2 - p^2)(k^2u^2 - p^2)}} - \int_{u^*}^{u_0} u^2 dH(u) \int_{p_1}^{p_2} \frac{pdp}{\sqrt{(w^2 - p^2)(u^2 - p^2)}}.$$

Here, the inner integrals are

$$K_1(w, u) = \int_{p_1}^{p_2} \frac{pdp}{\sqrt{(w^2 - p^2)(u^2 - p^2)}} = \ln\frac{\sqrt{w^2 - p_1^2} + \sqrt{u^2 - p_1^2}}{\sqrt{w^2 - p_2^2} + \sqrt{u^2 - p_2^2}},$$

$$K_2(w, u) = \int_{p_1}^{p_2} \frac{pdp}{\sqrt{(w^2 - p^2)(k^2u^2 - p^2)}}$$

$$= \ln \frac{\sqrt{w^2 - p_1^2} + \sqrt{k^2u^2 - p_1^2}}{\sqrt{w^2 - p_2^2} + \sqrt{k^2u^2 - p_2^2}}.$$

The final equation determining $H(u)$ is written in the form

$$g(w) = \int_{u^*}^{u_0} u^2 \{k^2 K_2(w, u) - K_1(w, u)\} dH(u). \quad (7)$$

Thus, we obtained the Fredholm first-kind integral equation for $H(u)$ satisfying the condition

$$dH(u) \ge 0. \quad (8)$$

The ray parameter p takes values in the interval $[p_1, p_2]$, where $p_1 = t'(x_{\min}) \ge 0$ and $p_2 = t'(x_{\max}) \le u^*$ or $p_1 = t'(x_{\max}) \ge 0$ and $p_2 = t'(x_{\min}) \le u^*$ if the time-distance curves are convex downward or upward, respectively. The ray parameter interval must be sufficiently large to provide a more stable determination of the wave velocity. To meet this condition, converted waves from earthquakes should be recorded in a large range of depths or epicentral distances.

Multiplying both sides of (5) by $1/\sqrt{w^2 - p^2}$ and integrating over p from p_1 to p_2, we obtain a system of equations similar to (7). After rearrangements, we find

$$f(w) = \int_{p_1}^{p_2} \frac{\Delta x(p) dp}{\sqrt{w^2 - p^2}}$$
$$= \int_{u^*}^{u_0} \{K_1(w, u) - K_2(w, u)\} dH(u). \quad (9)$$

The functions $f(w)$ and $g(w)$ can be written in the form convenient for numerical integration:

$$f(w) = f_P(w) - f_S(w), \quad g(w) = g_S(w) - g_P(w),$$

where

$$f_Q(w) = x(p_2)\arcsin\frac{p_2}{w} - x(p_1)\arcsin\frac{p_1}{w}$$
$$- \int_{x_1}^{x_2} \arcsin\frac{t'(x)}{w} dx,\ g_Q(w) = t(p_1)\sqrt{w^2 - p_1^2}$$
$$- t(p_2)\sqrt{w^2 - p_2^2} + \int_{x_1}^{x_2} \sqrt{w^2 - t'^2(x)}\, t'(x) dx,\ Q = \{P, S\}.$$

NUMERICAL SOLUTION OF THE PROBLEM

1. To solve equations (7) or (9) subjected to condition (8), we discretize them, dividing segment $[u^*, u_0]$ by N points into $N + 1$ partial segments $[u_j, u_{j+1}]$. We consider the function of jumps ΔH_j ($j = 1, 2, \ldots, N$) on the segment $[u^*, u_0]$. We have two systems of N equations that are linear with respect to N unknowns ΔH_j and are nonlinear with respect to the unknowns u_0 and u^*. These systems are written as

$$Ay - f = 0 \text{ and } By - g = 0, \quad (10)$$

where $A = \{a_{ij}\}$, $B = \{b_{ij}\}$; $f^T = \{f_i\}$; $g^T = \{g_i\}$; $y^T = \{\Delta H_j\}$; $i = 1, 2, \ldots, N$; $j = 1, 2, \ldots, N$;

$$a_{ij} = \left\{ \ln \frac{\sqrt{w_i^2 - p_1^2} + \sqrt{u_j^2 - p_1^2}}{\sqrt{w_i^2 - p_2^2} + \sqrt{u_j^2 - p_2^2}} - \ln \frac{\sqrt{w_i^2 - p_1^2} + \sqrt{k^2u_j^2 - p_1^2}}{\sqrt{w_i^2 - p_2^2} + \sqrt{k^2u_j^2 - p_2^2}} \right\}$$

for $i = N + 1, N + 2, \ldots, 2N$; $w_i = w_{2i}$ for $i = 1, 2, \ldots, N$;

$$b_{ij} = u_j^2 \left\{ k^2 \ln \frac{\sqrt{w_i^2 - p_1^2} + \sqrt{k^2u_j^2 - p_1^2}}{\sqrt{w_i^2 - p_2^2} + \sqrt{k^2u_j^2 - p_2^2}} - \ln \frac{\sqrt{w_i^2 - p_1^2} + \sqrt{u_j^2 - p_1^2}}{\sqrt{w_i^2 - p_2^2} + \sqrt{u_j^2 - p_2^2}} \right\}$$

for $i = 1, 2, \ldots, N$.

The unknowns ΔH_j must satisfy the linear constraints

$$\Delta H_j \ge 0. \quad (11)$$

It is obvious that, if u_0 and u^* are not known, systems (10) are underdetermined and, generally speaking, do not have a unique solution. We assume that u_0 and u^* are known.

We consider the functional

$$J = \|Ay - f\|^2. \quad (12)$$

The solution of the problem is accepted to be a vector y minimizing the functional J, under constraints (11). Thus, the P velocity distribution will be found on the condition that k (ratio of P- and S-wave velocities) is constant and known.

The minimization of functional (12), with linear constraints (11) and given u_0 and u^*, is a quadratic programming problem [Karmanov, 1975]. If both matrices A and B are nonsingular, such a problem has a unique solution, since functional J convex in y is bounded from below and is continuous on the convex set $Y = \{y | y \ge 0\}$ [Karmanov, 1975]. Quadratic programming problems are commonly solved with the use of a conjugate gradient method. This method always converges in a finite number of steps, but it requires an initial, or zero approximation [Nikasheva, 1968]. As a zero approximation, we can take the average seismic velocity in the layer and its thickness.

2. The seismic velocity in the layer and the refractor position can be found assuming that the *P*- and *SV*-wave velocities in the layer are constant. In this case, $n = 1$, $u_i = u$ = constant, $w_j = u$ = constant, and (7) and (9) are written in the form

$$f(u) = \{K_1(u) - K_2(u)\}\Delta H,$$
$$g(u) = u^2\{k^2K_2(u) - K_1(u)\}\Delta H. \tag{13}$$

Dividing the second equation by the first yields

$$\frac{g(u)}{f(u)} = u^2\frac{k^2K_2(u) - K_1(u)}{K_1(u) - K_2(u)}. \tag{14}$$

from which u can be found. Substituting u into any of equations (13), we obtain ΔH.

Equation (14) is easily solved by a simple iteration method, which, in the given case, converges to the solution at a virtually arbitrary initial approximation; i.e., the solution is sufficiently stable.

3. A principal point in solving our problem is the uniqueness of its solution, or more exactly, the elucidation of the conditions that provide a unique solution. As already mentioned, the same *P*-wave travel times are consistent with different velocity functions $v_P(z)$ having a measure $H(u)$. In such a case, the problem of uniqueness is reduced to the study of uniqueness or nonuniqueness of the solution to the system of nonlinear equations (10). In the general case, this problem is known to still be unsolved. In our case, we restricted ourselves to the study of numerical stability of the solution. We assume that our problem has a unique solution in the class of monotonic velocity functions, either decreasing or increasing. Then, the question of how to determine the integration limits in the right-hand sides of (7) and (9) arises. Following Burmin [1988, 1992] and, considering that the desired *P* velocity distribution in the layer is a function of the coordinate z: $u = u(z)$, with $u(0) = u_0$ and $u(z^*) = u^*$, we complete the definition of $u = u(z)$ so that the new function $\tilde{u} = \tilde{u}(z)$ coincides with $u(z)$ everywhere in the segment $[0, z^*]$, except, perhaps, a finite number of segment points. Namely, to retain the monotonicity of $\tilde{u} = \tilde{u}(z)$, $u = u(z)$ is defined at the ends of the segment $[0, z^*]$, setting $u_0 = u_{\max} \geq u(0)$ and $u^* = u_{\min} = p_2 \leq u(z^*)$. Here, $u_{\max}$ corresponds to the maximum index of refraction and to the value of $u(z)$ at the upper layer boundary $0z^*$ and can typically be always given *a priori*. The value $u_{\min}$ is not smaller than the slope of the P-wave traveltime curve at its right-hand end point; for example, if the crust is studied, it is not smaller than the slowness in the mantle. Evidently, the values of integrals (5) and (6) do not change in this case, and by virtue of the assumed uniqueness of the solution, the resulting distribution $\tilde{u}(z)$ differs from the exact solution only in zero-thickness layers. In the process of numerical solution of the problem, $u_{\max}$ and $u_{\min}$ are adjusted in an iterative way. This is effected as follows. In the first step, the problem is solved for the extreme values $u^{*0} = u_{\min}$ and $u_0^0 = u_{\max}$. The extreme values u_i and u_j for which ΔH_i and ΔH_j are nonzero (positive, with a given accuracy of calculations) are then taken as u^* and u^0, respectively. With the new extreme values $u^* = u_i$ and $u_0 = u_j$, the problem is solved once more. The procedure is repeated until the extreme values ΔH_1 and ΔH_N are nonzero.

4. The left-hand sides of (7) and (9) include the ray parameter p of the direct *P* wave, which is equal to the slope of the traveltime curve $t(x)$. The function $p = t'(x)$ can be determined in different ways, depending on the system of observations. In profile observations, only one seismic event is sufficient for determining the seismic velocity and localizing the refractor. If observations are made only at one point, it is necessary to record a sufficient number of events occurring at the same depth and at different epicentral distances. In both cases, we obtain observed traveltime curves. In the second case, the traveltime curve is always close to that consistent with a vertically heterogeneous medium. We do not specify here the notion of the closeness between traveltime curves, since it is not important in the given case. The experimental traveltime curve given, with uncertainties, by a discrete set of points must be smoothed by a curve convex upward or downward, depending on the considered branch of the refraction traveltime curve (before or after the inflection point; i.e., whether the direct or reverse branch is treated). Such a curve should be approximated by a convex cubic spline $T(x)$ [Burmin, 1980] that minimizes the functional

$$S = \|BT'' - y\|^2 \tag{15}$$

under the constraints

$$T_i'' \leq 0 \quad (T_i'' \geq 0), \tag{16}$$

where $i = 1, 2, \ldots, N$; T_i'' are the second derivatives of the spline $T(x)$ at points x_i; B is the $N \times N$ matrix, whose elements depend only on x_i; y is the vector of free terms depending on $t(x_i)$ at observation points x_i. The minimization of (15), with linear constraints (16), is also a quadratic programming problem [Karmanov, 1975].

To calculate the integrals in the left-hand sides of (7) and (9), it is necessary to determine the points of the *P* and *SV* traveltime curves, having identical values of the ray parameters p_1 and p_2. For this purpose, we can use interpolating formulas for the spline itself and its first derivative in terms of the second derivative of the spline at grid nodes [Alberg *et al.,* 1972].

5. To estimate the error in the derived refractor depth, we denote the actual depth as z^* and assume that

$\tilde{z}^*$ is the inferred depth and that the velocity in the layer $z^*\tilde{z}^*$ is constant. Then, for $\delta\Delta t = \Delta\tilde{t} - \Delta t$ and $\delta\Delta x = \Delta\tilde{x} - \Delta x$, at the condition $u(z) = u =$ constant, where $z \in [z^*, z^*]$, we have

$$\delta\Delta t = \left\{\frac{ku^2}{\sqrt{k^2u^2 - p^2}} - \frac{u^2}{\sqrt{u^2 - p}}\right\}\delta z,$$

$$\delta\Delta x = \left\{\frac{p}{\sqrt{u^2 - p^2}} - \frac{p}{\sqrt{k^2u^2 - p^2}}\right\}\delta z.$$

If the deviation of the difference between theoretical traveltime curves $\Delta t = \Delta t(x)$ from the experimental value $\Delta\tilde{t} = \Delta\tilde{t}(x)$ is taken as

$$\delta\Delta\tau(p) = \sqrt{[\delta\Delta t(p)]^2 + [p\delta\Delta x(p)]^2},$$

we find

$$\delta z(p) \le \sqrt{2\frac{u^2 - p^2}{u^4 + p^4}}\delta\tau(p). \quad (17)$$

This formula connects the determination error of z^* with the error $\delta\tau$ at a point of the traveltime curve corresponding to a ray with the parameter p. If the solution is sought on a segment of this curve, bounded by rays with parameters p_1 and p_2, then the error of z^* can be found as the average for all $p \in [p_1, p_2]$:

$$\begin{aligned}|\Delta z| &\ge \frac{1}{p_2 - p_1}\int_{p_1}^{p_2}|\delta z(p)|dp \\ &= \frac{1}{p_2 - p_1}\int_{p_1}^{p_2}\sqrt{2\frac{u^2 - p^2}{u^4 + p^2}}\delta\tau(p)dp \\ &\le \frac{\sqrt{2}|\Delta\tau|}{u^2(p_2 - p_1)}\int_{p_1}^{p_2}\sqrt{u^2 - p^2}dp \\ &= \frac{\sqrt{2}}{u^2(p_2 - p_1)}\left\{p\sqrt{u^2 - p^2} + u^2\arcsin\frac{p}{u}\right\}\Bigg|_{p_1}^{p_2}|\Delta\tau|,\end{aligned} \quad (18)$$

where $|\Delta\tau| \in [|\delta\tau|_{min}, |\delta\tau|_{max}]$.

It is readily seen from (18) that (a) the greater the range of ray parameters, the stabler the velocity distribution determined in the layer, and (b) the errors caused by the refractor departure from horizontality are proportional to the absolute value of the refractor inclination angle, because the determination errors of Δt and Δx increase in this case.

6. Now, we discuss the mathematical modeling results. As an example, the *P*- and *SV*-wave velocities in the crust were determined for a model close to the standard one. The theoretical traveltime curves of *P* and *SV* waves are presented in Fig. 1 in a 9 km/s reduction. These curves characterize waves traveling upward from a source located at a 250-km depth, and the conversion occurs at the Moho boundary, at a depth of 43.0 km. The slopes of the traveltime curves at their ends are equal to the respective reciprocals of apparent velocity values of ≈15.0 and 9.5 km/s. The crustal *P* velocities retrieved from the difference of *P* and *SV* traveltime curves are shown in Figs. 2 and 3. The initial velocity curve is obtained for a two-layer crust with a *P* velocity equal to 6.0 km/s in the upper layer and 6.75 and 6.84 km/s at the upper and lower boundaries, respectively, of the lower layer. The ratio of *P* and *S* wave velocities was set at 1.73. The values 4 and 8 km/s were taken as the bounds for the desired solution. This velocity interval was divided into 40 segments. The integrals in the left-hand sides of (7) and (9) were calculated using the Simpson formula.

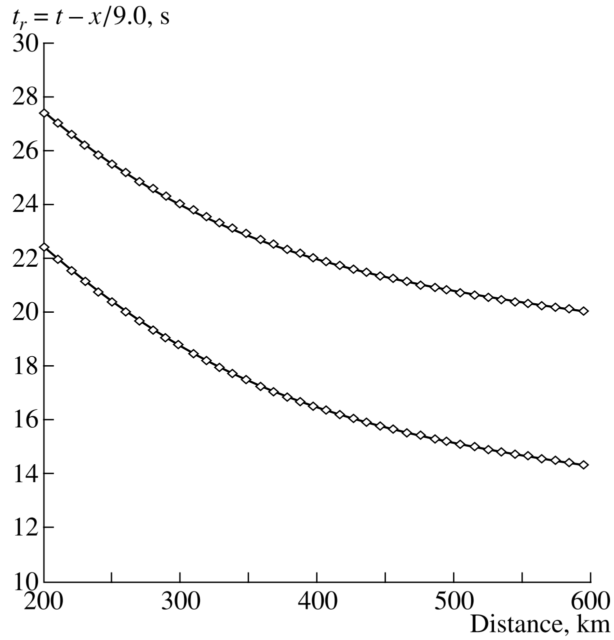

Fig. 1. Traveltime curves of direct *P* and converted *S* waves.

Figure 2 shows the velocity curve retrieved on the assumption that the *P* velocity in the entire crust is unknown. We see that the layer thickness and velocities are, on average, fairly well determined, although the inferred velocity curve by no means resolves the velocity jump within the layer.

Figure 3 presents the retrieved velocity profile on the assumption that the *P* velocity is known in the upper layer, but is unknown in the lower one. In this case, the results are close to those given in the model velocity distribution.

Figures 4 and 5 show the shortened traveltime curve and the related inferred velocity distribution in the

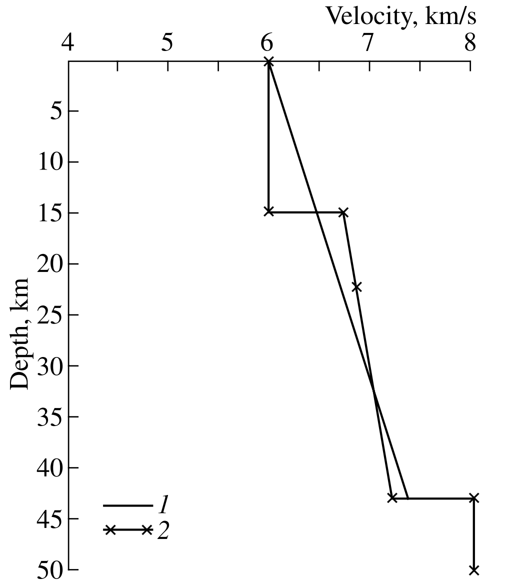

Fig. 2. Velocity profile in a two-layer crust model, reconstructed from the traveltime curves presented in Fig. 1. Shown are the (*1*) initial and (*2*) reconstructed velocity curves.

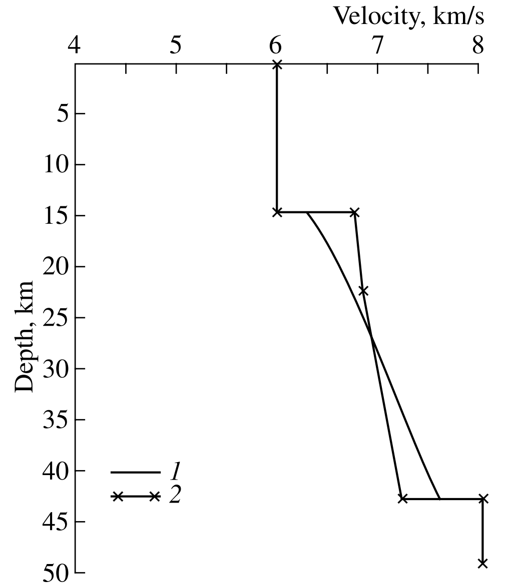

Fig. 3. Velocity profile in the lower layer of the two-layered crust model, reconstructed from the traveltime curves presented in Fig. 1. Shown are the (*1*) initial and (*2*) reconstructed velocity curves.

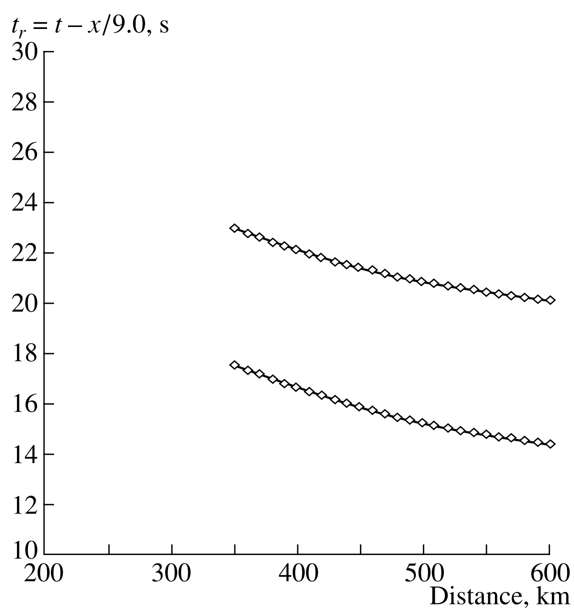

Fig. 4. Shortened traveltime curves of direct *P* and converted *S* waves.

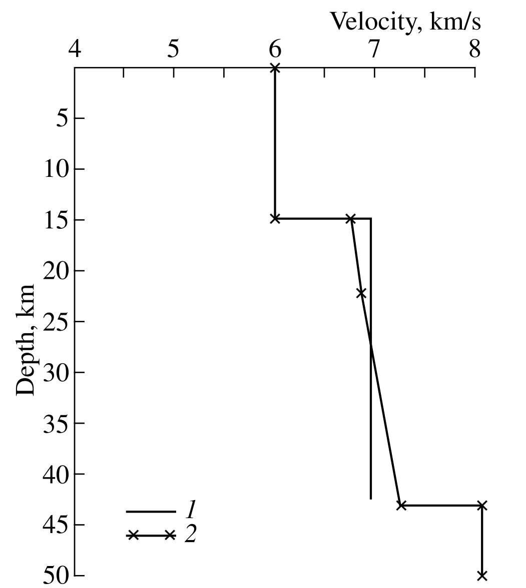

Fig. 5. The velocity profile in the two-layer crust model, reconstructed from the traveltime curves presented in Fig. 4. Shown are the (*1*) initial and (*2*) reconstructed velocity curve.

lower layer. The apparent velocities at the ends of this curve are 11.2 and 9.5 km/s. In this case, the inferred velocity curve has large errors, but the average velocity and the layer thickness are determined as stably as in the preceding case.

The numerical experiments also showed that, in order to reliably determine the layer parameters (velocity and thickness), the accuracy of the measured travel times must be not worse than 0.05 or 0.01 s on long (Fig. 1) or short (Fig. 4) traveltime curves, respectively. Otherwise, the solution of the problem becomes very unstable, which is mainly due to the insufficient accuracy of the apparent seismic velocity determination from the traveltime curves, or more specifically, due to uncertainties in the functions $x(p)$ and $t(p)$ derived from the *P* and *SV* traveltime curves.

Thus, based on the calculated results, one may state that the problem of the retrieval of velocity curve from the difference between the traveltime curves of converted *S* and direct *P* waves has a unique solution in the class of functions considered, i.e., monotonic positive step functions $H(u)$, because the solution does not depend on the chosen limits u_{max} and u_{min} (see above) and on the initial approximation $H_0(u)$. Nevertheless, the numerical solution of the problem calls for further study and the development of pertinent stable methods.

CONCLUSION

Notwithstanding the widespread opinion that the seismic wave velocity cannot be determined from records of body waves, we showed that both the average velocity and the vertical velocity distribution under the seismic station can be determined. For this, we derived the Fredholm first-kind integral equations relating the *P* wave velocity and the difference between converted *S* and direct *P* waves, provided that the ratio of the *P* and *SV* waves in the layer above the refractor is constant and known.

In order to obtain the *P* and *SV* traveltime curves, it is sufficient to record earthquakes with the same hypocenter depths at a single point, but at different epicentral distances. The greater the range of ray parameter variation, the stabler the inferred velocity distribution in the study layer. The traveltime curves derived from such observations are adequate to a vertically heterogeneous medium only under the observation point. Given a sufficient number of such points, it is possible to construct the three-dimensional velocity distribution in the layer.

The principal stumbling block in the determination of layer parameters from the difference between *P* and *PS* arrival times is the requirement to find these times with an accuracy no worse than 0.01 s. This problem is not new and is topical in solving a number of velocity and amplitude inversion problems of seismology. For example, Nolet [1990], in his seismic tomography review, states that, in order to arrive at an adequate solution of tomography problems, it is more advantageous to increase the accuracy of measurements rather than the volume of data, because the data accuracy controls, to a large extent, the number of iterations and actually governs the convergence of various methods. Our situation is quite similar, although the solution of the problem can probably be stabilized at the stage of determining the functions $x(p)$ and $t(p)$ for *P* and *SV* waves.

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