On reconstruction of the three displacement vector components from SAR LOS displacements for oil and gas producing fields

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Abstract

SAR interferometry estimates Line-Of-Sight displacements of natural or man-made reflectors. To estimate all three components (e.g. north, east and up) of the displacement vector, it is necessary to use additional data or physical models of causative sources of the observed displacements. We propose a method to reconstruct the three components of the displacement vector at oil and gas exploration fields, based on an assumption that the registered LOS displacements are caused by a decrease or increase in seam pressure due to hydrocarbon extraction or fluid injection. We present an example of an evaluation of LOS displacements and the reconstruction of the three displacement vector components using SAR images for the Romashkino oil field near Almet’evsk city (Tatarstan Republic, Russian Federation).

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1. Introduction

Monitoring surface displacements at oil and gas exploration fields is a useful tool not only for registration hazardous deformations of buildings and infrastructure facilities but it also helps to monitor reservoir pressure, trace hydrocarbon and injected fluid migration.

Besides measuring surface displacements with GNSS systems (GPS or GLONASS), spirit levelling and other geodesy techniques, synthetic aperture radar (SAR) interferometry has been used often in recent years. One of the first studies was performed by van der Kooij et al. [1] in which the subsidence of the Groningen gas field was estimated. Based on analysis of pairs of SAR images for the Lost Hills and Belridge oilfields, Fielding, Blom, and Goldstein [2] discovered areas with average rates of subsidence of more than 40 cm/year. Patzek and Silin [3] presented design of a multilevel, integrated surveillance and control system which uses InSAR estimated displacement at producing field surface. Persistent Scatterers method for the oil and gas fields was for the first time applied in [4]. There is extensive literature devoted to the application of SAR images for monitoring of producing fields, which we will not cover here as it is not the purpose this paper.

It is important to note that geodesy and InSAR measurements contain errors of different kinds. In particular, both GNSS and SAR interferometry data retain atmospheric artefacts. The accuracy of SAR interferometry estimates depends on accuracy of the digital elevation (DEM) model used, magnitude of spatial and temporal baselines, precision of orbital data, and changes of surface reflectivity between image performance (for example, due to the dynamics of vegetation or snow cover). To develop an efficient strategy for InSAR image processing and suppression of nuisance signals, joint analysis of all available monitoring data is desirable. The problem of fusion of different data is that they are acquired at different surface points, at different times, with varying accuracy, and importantly, they measure different characteristics of the displacement vector. GNSS most accurately records horizontal displacements to the north and east. Analysis of pairs of InSAR images (DInSAR) or image series (PSInSAR) provides estimates of displacements in projection on the satellite’s line-of-sight (LOS) direction. Repeated levelling registers vertical displacements relative to a benchmark. As a result, direct comparison of these data is very difficult, if at all possible. To illustrate this point, consider a simple example of the Earth’s surface displacement as a result of a pressure increase in a small spherical volume, located at 1 km depth (Fig. 1). Shown in Fig.1, LOS displacement - $U_{\text{LOS}}$ is written [5]:

$$U_{\text{LOS}} = U_x \cos \theta - \sin \theta \cdot (U_x \cos \beta + U_y \sin \beta),$$

where: $U_x$, $U_y$, and $U_z$ are displacements to the north, east and vertical (positive upward), $\theta$ is incidence angle, $\beta$ stands for satellite heading.
Fig. 1. Surface displacements (nondimensional) as a result of pressure increase in a small volume at 1 km depth. $U_n$, $U_e$ and $U_z$ - are displacements to the north, east and vertical (positive downwards). $U_{LOS}$ - LOS displacement (positive to satellite) is calculated from equation (1) from ENVISAT descending track with a heading of 197°, and an incidence angle of 20.4°. LOS azimuth is 287°. Negative values are shown in gray scale. Horizontal and vertical axis (latitude and longitude in degrees) are scaled to keep equal linear distances.

Fig. 1 shows that maximum vertical displacement is at the point where both horizontal components are equal to zero. To locate this point, a net of GNSS stations has to be rather dense. Provided by SAR interferometry $U_{LOS}$ displacement distribution is not isometric: its maximum is shifted to the east from the point of maximum vertical displacement and there is an area of $U_{LOS} < 0$ as well. If one indiscreetly suggests that both horizontal components of the displacement vector are negligibly small, and estimates vertical displacement using eq. (1) as $U_{LOS}/\cos(\theta)$, then a wrong conclusion about non-isometric uplift and presence of a subsidence area is made. Repeated levelling provides vertical displacements, hence to locate the point of maximum uplift a dense net of benchmarks is necessary. In addition, each benchmark experiences a horizontal shift which cannot be registered by levelling.

Combining horizontal displacements $U_n$, $U_e$ at GNSS stations with $U_{LOS}$ (provided by SAR interferometry), $U_z$ can be found from equation (1). The problem with this approach is that in favourable conditions SAR interferometry identifies several tens or hundred thousands of persistent scatterers (PS) within one image. GNSS networks are never dense enough for conversion of all PSs.
If SAR images from both ascending and descending tracks are available, one obtains two equations with different \( E \) and \( T \) for three unknown displacement vector components. As heading \( E \) usually is close to 0 or \( 2\pi \), multiplier \( \cos(E) \) at \( U_e \) in equation (1) is small, which allows \( U_e \) to be neglected and two other components can be determined from the two equations for ascending and descending tracks. For this to be possible, the suggestion that \( U_e \) is not much exceeding \( U_n \) and \( U_z \) should be supported by other data.

The most efficient solution of the problem is based on modelling causative processes of registered displacements. For landslides and glaciers, a possible assumption is that they move against the topographic gradient vector. In this case, data from two satellite tracks are necessary to solve the problem. When LOS displacements are registered from one satellite track, additional assumptions are necessary e.g. that the displacement vector has the same dip as topography gradient vector. For coseismic and post-seismic displacements, the fault surface can be approximated by several planes; their parameters and along strike and dip displacements can be determined from the corresponding inverse problem solution (for detailed overview see e.g. [6]). For oil and gas exploration fields, a possible assumption is that the main part of displacements is associated with a decrease or increase in seam pressure due to hydrocarbon extraction or fluid injection. Let us consider a method for combining data based on this hypothesis.

2. Reconstruction of three components of displacement vector for oil and gas producing fields

The method to calculate the Earth's surface displacement caused by change of pressure in a certain subsurface volume is as follows. First, subdivide the volume (possibly an accumulation of several unrelated domains) into a set of small elementary volumes. Extraction and injection of fluids can cause pressure to drop or increase in each of these elementary volumes. The upper crust response to relatively short production processes can be modelled by an isotropic elastic half-space. As this model is linear, the solution for the total volume is the sum of solutions for all the elementary volumes. When the media can be considered homogeneous, the problem for the elementary volume has an analytical solution. Note that the solutions for a homogeneous elastic half-space and for the viscoelastic media are exactly the same even if the coefficients for the viscoelastic media are time-dependent [7]. Consider a half-space \( z \geq 0 \) where top \( z = 0 \) is a free surface. Stress within the model results from extension or contraction of the small elementary volume \( \varepsilon_e(\vec{r}) \), where \( \vec{F} \) is a radius-vector directed from the volume centre to a current point within the half-space. The solution for the half-space is built upon the solution for infinite space [8], where the displacement field \( u(\vec{r}) \) is:

\[
4\pi u(\vec{r}) = -\nabla \varphi, \tag{2}
\]

when potential \( \varphi \) obeys Poisson equation: \( \Delta \varphi = -4\pi \frac{1-v}{1-\nu} \varepsilon_e \) and \( \nu \) is Poisson's ratio. Hence, the solution for equation (2) is written:

\[
\varphi(\vec{r}) = \frac{1+v}{1-\nu} \int \varepsilon_e(\vec{r}') \frac{d\vec{r}'}{|\vec{r}' - \vec{r}|}, \tag{3}
\]

where the integral is performed over all extensional and compressional domains in half-space \( z > 0 \).

The solution for the half-space is determined in [9]. It shows that the displacement field at free surface \( z = 0 \) of a half-space is equal to solution for infinite space multiplied to \( 4(1-\nu) \) (see [10] for detailed consideration). For an isolated centre of expansion or construction situated at point \( \vec{r}_0 = (x_0, y_0, z_0) \), strain is equal to \( \varepsilon_e(\vec{r}) = \delta(\vec{r} - \vec{r}_0) \) and the components of the displacement vector are written as:

\[
U_x(x_i, y_i, 0) = \frac{(1+v)S}{\pi} \frac{x_i - x_j}{R^3}, \quad U_y(x_i, y_i, 0) = \frac{(1+v)S}{\pi} \frac{y_i - y_j}{R^3}, \quad U_z(x_i, y_i, 0) = \frac{(1+v)S}{\pi} \frac{z_j}{R^3}, \tag{4}
\]

where \( R^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + z_j^2 \), \( z_i = 0 \).
To find the components of the displacement vector on free surface \( z = 0 \) caused by pressure changes in a volume \( V \) (possibly an accumulation of separate domains), it is necessary to integrate functions (4) over this volume. Since the amount of expansion/contraction is not constant over the volume \( V \), coefficient \( S \) becomes a function of the spatial coordinates. Finally, using formula (1) LOS displacement equals:

\[
U_{\text{LOS}}(x,y,0) = \frac{(1+\nu)}{\pi} \int_V S(\xi,\eta,\zeta) \left[ (z-\xi)\cos \theta - \sin \theta \ast (y-\eta)\cos \beta + (x-\xi)\sin \beta \right] \left( (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2 \right)^{-3} d\xi d\eta d\zeta ,
\]

where the integration variables \((\xi,\eta,\zeta)\) run over the total volume \( V \).

Equations (4), up to the coefficients, coincide with those for derivatives of the gravitational potential. In the case of gravity, the factors are mass and gravitational constant. For elasticity problem, equation (4) includes Poisson’s ratio and volume changes \( S \). Change of the elementary volume depends on compaction coefficient and pressure change [11]. This analogy permits to derive analytical expressions for the surface displacements caused by pressure variations in volumes such as cylinder, rectangular parallelepiped, a semi-infinite layer, etc., and also to apply for the analysis and solution of related problems the well-developed potential theory.

To reconstruct the three components of the displacement vector one should find \( S(x,y,z) \) from equation (5) and then substitute it in equations (4). The problem to find \( S(x,y,z) \) in the general case is not unique, and even in specific classes of uniqueness, it has no stability and calls for different methods of regularization [12]. A possible way to solve the problem is to construct a model of the exploration field under study. For this, it is necessary to know the geometry of the productive formation, distribution of formation pressure, porosity, permeability, hydrocarbon saturation, well location, exploration depth intervals, periods and volumes of extraction or injection, etc. After all required information is assigned, all three components of the displacement vector at the Earth’s surface, and LOS displacements can be calculated. Then, an inverse problem can be stated: to determine or refine a model parameter under the condition of best fit of the calculated surface displacements and all available monitoring data. Reliability of the solution depends on the accuracy with which the geological structure and physical properties are known and assigned. In addition, solving this problem is time-consuming.

Another solution, which we construct here is based on the fact that the displacement field in an elastic half-space, as a result of pressure changes in its domains \( V \), obeys the Laplace equation outside \( V \) where \( S(x,y,z) = 0 \). Hence \( U_r, U_\theta \), and \( U_z \) are harmonic functions. From the Stokes’ theorem, it follows that if a function is harmonic in some space \( R \) (in particular in the upper half-space \( z \leq 0 \)), this function and all its derivatives and transforms (e.g. \( U_{\text{LOS}} \)) can be uniquely determined by the values on the boundary of this space (in particular at \( z = 0 \)), using the solution of corresponding Dirichlet, Neumann or mixed boundary-value problem (e.g. [13]). Hence if \( U_{\text{LOS}} \) is assigned at \( z = 0 \), one can find all displacement vector components without knowledge of the true geological structure, physical parameters, steam/injection performance etc.

An efficient way for finding \( S(x,y,z) \) from equation (5) is by equivalent source approximation [14]. To choose the type of equivalent sources and their position, an analogy between equivalent source approximation and optimal Kolmogorov-Wiener filtering can be used [15]. Let us assume that a gravity anomaly is the sum of two uncorrelated components of different wavelength. We represent the regional component by a cumulative gravity field of a set of elementary sources (e.g. point masses or dipoles), situated below the data points at some depth \( H_d \). The local component can be represented as a gravity field of another set of sources located in the same horizontal positions, but at depth \( H_l < H_d \). The source masses are determined from the equivalence of the total model field to the given one. If: (1) the autocorrelation function of each component is equal to the gravity field of elementary source situated at corresponding depth, and (2) the ratio of masses of the two sources situated below every data point is constant and equal to \( \rho \) – the ratio of the root-mean square amplitudes of the corresponding signal components, then the process of equivalent sources approximation is identical to that of Kolmogorov-Wiener optimal filtering.

3. Method of solution of the general problem

Because measurements by different methods cover different time intervals, we will further refer to \( U \)-values as average displacement rates, assuming that these values characterize the same time interval or that \( U \) does not change.
when time intervals are close. Therefore, we will further refer to $S$-values as average rates of elementary volume contraction or expansion. Suppose that GNSS repeated measurements provided average displacement rates (in mm/year) to the north and east at $k$ points $\{\lambda_i, \varphi_i, U_{x_i}, U_{y_i}\}$, $i = 1, 2, \ldots, k$. Suppose that SAR interferometry identified $m$ PSs and their average displacement rates are $\{\lambda_j, \varphi_j, U_{LOS,j}\}$, $j = 1, 2, \ldots, m$. Suppose also that along $n$ repeated leveling profiles the vertical displacement rates are $\{\lambda_p, \varphi_q, W_q\}$, $p = 1, 2, \ldots, n$, $q = 1, 2, \ldots, m_p$, where $m_p$ is the number of points on the profile with index $p$. Vertical displacements on each profile are usually defined relative to a reference point. Each profile can have its own reference point.

To solve the problem and to separate long- and short-wavelength components of the displacement fields we used small extending or contracting elementary volumes $S_{j j}$ situated below each PS at two different constant depths. To treat jointly all available data, we determined $S_{j j}$ under following condition:

$$
\min_{W_{q}} \left( \sum_{i=1}^{k} \left( U_{x_i} - U_{LOS_{x_i}} \right)^2 + \sum_{j=1}^{m} \left( U_{y_j} - U_{LOS_{y_j}} \right)^2 + \sum_{p=1}^{n} \left( U_{z_p} - U_{LOS_{z_p}} \right)^2 \right)
$$

(6)

Here $\sigma_{SAR}$, $\sigma_{GNSS}$ and $\sigma_{lev}$ stand for estimates of mean square errors in data of SAR interferometry, GNSS geodesy, and repeated levelling. Superscript $calc$ stands for corresponding calculated component. Average vertical displacement rate calculated for the benchmark of each profile has to be subtracted from all calculated vertical displacement rates $W_{q,calc}$ on the corresponding profile.

The mathematical problem to determine $S(x_j, y_j, H_q)$ from eq. (6) reduces to solving a system of linear equations. After $S(x_j, y_j, H_q)$ are found, all components of displacement vectors can be calculated in points of measurement or at a regular grid for more detailed study. Components of stress and strain tensors, direction of principal axis, surface tilt and other values used for monitoring of geodynamic hazard, can be calculated as well.

4. Example of application of the method to the Romashkino oil field

This producing field is situated in the central part of the East European Platform in Tatarstan Republic. Geodynamic activity in the area is mostly related to karst processes and oil field development. Production of hydrocarbons from wells results in technogenic seismicity and surface deformations. Beginning in the 1980’s, seismological observations were initiated, and since 1991, repeated precision levelling has been in place. Unfortunately, periods of repeated measurements do not coincide with periods covered by SAR acquisitions, so we demonstrate the method using only SAR data.

To estimate surface displacements, we used Envisat SAR images from ascending track 49 for acquisition period from 04.2004 to 10.2005. Three of eleven available images were rejected because of low coherence. Images were acquired in months 04, 07, 10, 12 of 2004 and 02, 08, 09, 10 of 2005.

Average LOS velocities were approximated by small spherical volumes situated at two levels: 0.4 and 1.0 km below each PS. The ratio of lower to upper volume changes was 1:100, so the upper level mostly approximated low amplitude short-wavelength noise.

Fig. 2a shows average displacement rates for the period covered by SAR images for a small area of 0.1°×0.1° around Almet’evsk city. Calculations were performed using StaMPS software [16], based on the method developed in [17]. The most prominent feature of the average LOS displacement rate is a sharp boundary between negative and positive values running along the southern boundary of the city, between residential and industrial zones. Average LOS velocities $U_{LOS}$ were transformed into the three velocity vector components, of which the vertical one is shown on Fig. 2b. At first glance, these schemes are similar, but more detailed analysis of cross-sections along profiles (Fig.3) and velocity maps (Fig.4) reveals important differences.
Fig. 2. The average surface displacement rates at the Romashkino oil fields around the Almet’evsk city (situated in the centre of the figures at the cross-section of the geographic lines 54.9N and 52.3E) for the period 04.2004 – 10.2005. (a) – the average LOS velocities. (b) – the calculated average vertical displacements.

The main differences are as follows. First, as was expected, the maximum amplitude of the displacement velocity vector is approximately 1.5 times more than its projection in the direction of LOS. Second, in profile a (Fig. 3), running from west to east, there is a regional skew in the displacement field of the satellite. Because of the displacement of anomalies in LOS projection, some parts of the curves differ significantly. It is important to bear in mind that in areas where vertical and the LOS displacement greatly differ, horizontal displacements are considerable.

Maps of the LOS and vertical average displacement rates (Fig.4) were built from a dense grid calculated using estimated $S(x, y, H_n)$ values. In area 1 on $U_z$ map of Fig.4, the subsiding area is bounded by an area of uplift marked as 2. Area 3 stretches to SE as a single uplifting belt which abruptly changes to subsidence area 5 at its southern boundary. Local uplifts at 4 within area 5, is also evident. Some of these features are not clearly seen on the map of $U_{LOS}$.

Plots in the lower line of Fig. 4 show horizontal components of displacement rate to east and north. Average rates of eastward displacements exceed 10 mm/year, when northward displacements mount to 14 mm/year. Hence neglecting horizontal displacements will lead to considerable errors in vertical component estimates. This is evident also from the scheme of vectors of average horizontal displacement rate shown in Fig. 4e.

Fig. 3. The average rate of the LOS (dashed line) and vertical (continuous form line) displacement rates along (a) east-west trending profile and (b) south-north trending profile. The position of the profiles is shown by the arrows in Fig. 4a.
5. Conclusions

The method to calculate all three components of the displacement vector using SAR interferometry and LOS displacements for producing oil and gas fields, is presented. The method is based on the equations that define the displacements on the free surface of an elastic or visco-elastic half-space as a result of pressure changes in the volumes of a given configuration (in this paper small spherical volumes were used). This results in an efficient method for calculating a numerical solution.

The proposed method is suitable for finding a displacement field consistent with any data of repeated measurements (GNSS, levelling, SAR interferometry). The method is demonstrated on the surface displacements around Almet’yeysk city on the Romashkino oil field. Calculated vertical displacements differ from LOS displacements both in morphology and in magnitude. Average horizontal displacement rates reach 12 mm/year. The results show that the assumption that the horizontal components are negligibly small could lead to significant errors.

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