ISSN 1028-334X, Doklady Earth Sciences, 2009, Vol. 428, No. 7, pp. 1202–1207. © Pleiades Publishing, Ltd., 2009.
Original Russian Text © Yu.L. Rebetskii, 2009, published in Doklady Akademii Nauk, 2009, Vol. 428, No. 3, pp. 397–402.

GEOPHYSICS

Estimation of Stress Values in the Method of Cataclastic Analysis of Shear Fractures

Yu. L. Rebetskii

Presented by Academician S.A. Fedotov March 25, 2009

Received April 14, 2009

DOI: 10.1134/S1028334X09070368

The possibility of estimation of tectonic stress values through the method of cataclastic analysis (MCA) of shear fractures [1] appeared after the development of its algorithm of the second stage of reconstruction, oriented at calculation of relative values of spherical and deviator components of stress the tensor. After the first stage of this method, through the data on a group of focal mechanisms from a homogenous sample of earthquakes, three Euler angles are calculated, which determine the orientations of the three main axes of the tensor, and the value of the Lode–Nadai coefficient μ_σ, which characterizes the type of stress tensor. Such a homogenous sample of earthquakes defines the quasi-homogenous stage of deformation for a calculated domain of the Earth's crust. Within the framework of the second stage of MCA, in the Mohr diagram we analyze the distribution of points that are characterizing the values of the stress vector at the shear plane for earthquakes from this homogenous sample, and this analysis allows us to estimate the relative values of maximal tangential stress $\left\langle \frac{\tau}{\tau_f} \right\rangle$ and effective pressure difference $\left\langle \frac{p^*}{\tau_f} \right\rangle$ ($p^* = p - p_{fl}$ is the pressure difference in solid rocks and the fluid pressure in fractural-porous space). Valuation of p^* and τ is made at the unknown value of the effective internal adhesion τ_f of rock massifs. The algorithm of such an estimation in MCA is based on an approximation at the Mohr plane (effective normal stress σ^*_{nn} tangential with a stress τ_n) of the diagram strength of fractured rocks with the help of two limit lines (Fig. 1); the upper one characterizes the limit of effective internal adhesion of massifs, and the lower one corresponds to the minimal resistance of static friction for the already existing fractures and ruptures. For smoothing scales on a megascopic level (more than the first few meters of smoothing), let us consider that the coefficients of internal and surface static pressure are equal ($k_f = k_s$).

Schmidt Joint Institute of Physics of the Earth, Russian Academy of Sciences, Moscow

ALGORITHM OF STRESS VALUE ESTIMATION ON THE BASIS OF DATA ON STRESSES DISCHARGED IN FOCI OF STRONG EARTHQUAKES

To estimate the absolute stress values within the framework of the third stage of MCA, it is suggested to use the data on the value of tangential stress discharged in the focus of a strong earthquake. These data are the result of analysis of seismic records on the basis of a complex of seismologic methods, and thus, within the

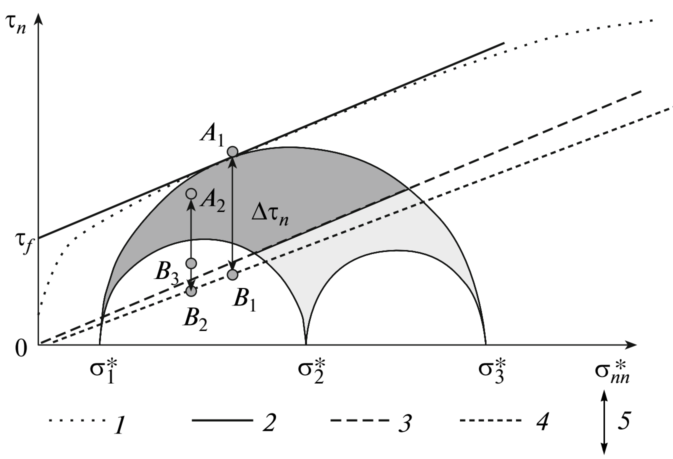

Fig. 1. Geometric analysis of stresses discharged in the earthquake focus in the Mohr diagram. (*1*) A possible view of the curvilinear envelope, which characterizes the limit of effective internal strength for the rock massif, corresponding to the level of reconstructed stress parameters; (*2*) linear approximation of the limit curvilinear envelope used in the MCA; (*3*) the line of dry static friction of rocks; (*4*) the line of dry kinematic friction of rocks; the dark gray area is the geometrical position of stress points on the fracture plane, which may be activated for this stress tensor (the light gray color marks the other stressed states at arbitrary oriented planes); (*5*) the value of discharged tangential stresses.

framework of the MCA, they may be considered as additional ones, forming the basis of the third stage of reconstruction in tensor stress parameters. Such an approach became possible because, after the first two stages of reconstruction of stress parameters, MCA allows us to estimate the relative value of discharged stresses in foci of strong earthquakes, whose characteristic linear size is not less than one of the smoothing window of the reconstructed stress tensor parameters. The algorithm of such an estimation is illustrated in Fig. 1, where in the Mohr diagram the vertical segment connecting the tangency point of the limit envelope with the big Mohr circle of the stressed state in the area, including the focus of occurred earthquake, is shown with line of kinematic friction. The length of this segment (A_1B_1) is equal to the difference value of tangential stresses before and after an earthquake, i.e., to the discharged stress ($\Delta\tau_n = |\tau_n^0 - \tau_n^1|$), in the case if the direction of sliding in the focus coincides with the direction of tangential stress, acting there before earthquake. The possibility of such a geometric interpretation is based on the fact, arising from the theory of destruction [2], that normal stresses at the plane of continuity rupture if the shift type before and after its activation are equal ($\sigma_n^0 = \sigma_n^1 = \sigma_n \le 0$), and also on the equity of tangential stresses at the rupture after its activation to the value of kinematical friction ($\tau_n^1 = -k_k\sigma_n^*$). Let us write the expression for the value of discharged stresses, valuated at τ_f:

$$\left\langle\frac{\Delta\tau_n}{\tau_f}\right\rangle = \bar{\tau}_n\left\langle\frac{\tau}{\tau_f}\right\rangle + k_k\left(\bar{\sigma}_n\left\langle\frac{\tau}{\tau_f}\right\rangle - \left\langle\frac{p^*}{\tau_f}\right\rangle\right) \quad (1)$$

with

$$\bar{\tau}_n = (1-\mu_\sigma)l_{1n}l_{1s} + (1+\mu_\sigma)l_{3n}l_{3s} > 0,$$

$$\bar{\sigma}_n = (1-\mu_\sigma)l_{1n}^2 - (1+\mu_\sigma)l_{3n}^2 + \frac{2\mu_\sigma}{3} < 0,$$

where $\bar{\tau}_n$, $\bar{\sigma}_n$ are reduced deviator stresses at the rupture plane, l_m are direction cosines of the normal to the focus plane of the analyzed strong earthquake with axes of main stresses σ_i ($i = 1, 3$), and k_k is the coefficient of kinematic friction. In the MCA algorithm, implemented in calculations below, the value of static dry friction k_s and effective internal friction k_f were assumed at 0.6, and the value of kinematical friction k_k was 0.5. The possibility of using expression (1) implies the constancy of fluid pressure in each of the calculated domains at the moment of rupture activation ($p_{fl}^2 = p_{fl}^1$). The first summand in (1) characterizes the values of tangential stresses τ_n^0, acting along the plane of focus before the earthquake (point A_1 in Fig. 1), and the second one does the tangent of stresses τ_n^1, which will act here after the earthquake (point B_1 in Fig. 1). All the parameters in expression (1) for determining the relative value $\left\langle\frac{\Delta\tau_n}{\tau_f}\right\rangle$ can be obtained after the first two MCA stages. It should be noted that use of expression (1) requires knowledge of the realized plane of the earthquake focus, i.e., one of the two nodal planes to be chosen, which is usually done only for strong earthquakes.

After the calculation of the relative value $\left\langle\frac{\Delta\tau_n}{\tau_f}\right\rangle$ by the data of the first two stages of MCA, we might estimate τ_f. As additional data, we use the value of stresses $\Delta\tau_n$ discharged during the strong earthquake:

$$\tau_f = \frac{\Delta\tau_n}{\langle\Delta\tau_n/\tau_f\rangle}. \quad (2)$$

The value $\Delta\tau_n$ on the right of (2) is defined with the use of data on the scalar value of seismic moment M_0 and from the geometry of the focus [3]:

$$\Delta\tau_n = \frac{\chi M_0}{WL^2}, \quad (3)$$

where L and W are the corresponding length (the greatest of the subhorizontal sizes) and the width of the focus, and χ is a parameter dependent on the type of displacement in the focus (fault, upthrust, or shift) and on its geometrical form ($0.65 \le \chi \le 1.85$).

It is shown in [4] that if the energy associated with change in the area of the growing rupture surface is negligible and we consider that tangential stresses after the beginning of displacement along the rupture have not changed, but remained equal to the dry kinematic friction, then tangential stresses discharged in the focus could be defined using the data on released seismic energy E_s:

$$\Delta\tau_n = \frac{2\mu E_s}{M_0}, \quad (4)$$

where μ is the modulus of elastic shift (for the crust μ is $\approx 3–5 \times 10^{10}$ MPa); E_s and M_0 are presented in joules.

It should be noted there that there is a proportion in seismology that allows us to estimate the so-called seeming stress $\bar{\tau}_n = \frac{|\tau_n^0 + \tau_n^1|}{2}$, which is the average stress between the two states at the rupture, on the basis of data on seismic energy:

$$\eta\bar{\tau}_n = \frac{\mu E_s}{M_0}, \quad (5)$$

where $\eta = \frac{\Delta\tau_n}{2\bar{\tau}_n}$ is the so-called seismic efficiency. Expression (5) is often used for definition of $\bar{\tau}_n$. But, due to the great uncertainty of the η value, such estimations cannot consider reliable.

The value E_s in expression (4) could be obtained using correlation dependences with the magnitude of earthquake M_e, which characterizes energy radiated in seismic waves:

$$\log E_s = \frac{3M_e}{2} + 4.35. \quad (6)$$

If the destruction is implemented along the previously existing rupture, which has not restore its adhesion strength to the undisturbed state ($\tau_s < \tau_f$), then in the Mohr diagram the vertical segment, which characterizes discharged stress, begins from the point lying below the limit envelope, but higher than the line determining the minimal stress of dry static friction (the segment A_2B_2 in Fig. 1). In the case when the direction of displacement along the rupture does not coincide with the direction of tangential stresses acting on the plane before activation, the vertical segment does not reach the line of kinematical friction resistance (the segment A_2B_3 in Fig. 1).

The stress parameters substituted into expression (1) should correspond the smoothing period, which precedes the strong earthquake, and data on its discharged stresses will be used for determining τ_f. For example, using the catalogue of earthquake focal mechanisms with the range of magnitudes of 4.5–7 allows us to tell that the reconstructed parameters of the stress tensor may correspond to the smoothing scale of 20–100 km (the concrete value depends on the density of earthquakes foci distribution), and the range of magnitudes of 2.5–6 may correspond to the smoothing scale of 5–30 km. For these scales of stress smoothing, strong earthquakes, data on whose discharged stresses can be used for estimation of internal adhesion, should not have magnitudes less than 6.5 and 5.5 correspondingly, i.e., the focal area should not be less than the stress smoothing window.

The algorithm of the third stage of MCA allows us to estimate the value of effective internal adhesion of rocks τ_f, and then, using the data on relative values of maximal tangential stresses and effective pressure (the second stage of MCA), transit to their absolute values.

EXAMPLES OF ESTIMATION OF THE VALUE OF EFFECTIVE INTERNAL ADHESION OF ROCK MASSIFS

For calculation of τ_f, we will use the data on values of discharged stresses of the Tokachi Oki earthquake (TOE) with M_w = 8.3, which occurred southeast of Hokkaido Island September 25, 2003, at a depth of 27 km, and of the Simushir earthquake (SE), which occurred November 15, 2006, with M_w = 8.3 (the focus depth was 28 km according to Tikhonov [5] and 30 km by the data of the United States Geological Service [6]).

Reconstruction of stresses in the preparation of TOE was carried out on the basis of seismological data on focal mechanisms of earthquakes from the catalogue of the Japanese Meteorological Agency [7]. The fullest range of earthquake magnitudes from this catalogue for the studied area of the crust is $3.5 > M_w > 5.5$ (85%), and the distribution density of their foci allowed us to tell about the smoothing scale of stress parameters of 5–20 km. The reconstruction was made in mesh points of the $0.1° \times 0.1°$ grid for the layer with the median surface at a depth of 20 km. According to the data of [6] and the aftershock sequence [8], the focus corresponded to the gently tilted plane (7°) with the strike azimuth 234°, whose length along the oceanic hollow is L = 180 km and the length across is W = 150 km, while the depth drop is 10–30 km. In Fig. 2a the values of discharged tangential stresses in foci of 118 domains in the neighborhood of the TOE focus, valuated at τ_f, are shown, for which the data on components of stress tensors were obtained by the result of the first two stages of reconstruction. MCA [1] has the possibility to choose one of the two nodal planes as the plane implemented in earthquake. Such an analysis is made in the Mohr diagram and allows us to determine the nodal plane, which is close by orientation to the plane of rocks spalling. In particular, for the studied earthquake, the gently tilted nodal plane is presented in the Mohr diagram (Fig. 2) as the pentagon close in its orientation to the spalling plane, while the steep plane (sinking angle is 83°, strike is 41°) enters the area forbidden for destruction (triangle). Thus, the criterion of determining of nodal plane occurring in the earthquake focus used in MCA gives the same result as seismological methods, in which the focus is not considered as a point source of seismic waves.

Calculation of the average value $\left\langle \frac{\Delta\tau_n}{\tau_f} \right\rangle$ for 118 domains (Fig. 2a) has given the result 0.78. The areas of these 118 domains are about 65% of the earthquake's focus area. The absence of data on stress parameters in other parts of the focus is related to the low level of seismic activity there. It indirectly evidences that the stress state of these parts was not critical and, consequently, the stresses discharged here might be small in their value. Let us consider further the contribution in released seismic energy for parts where the data on stress parameters are absent as $\left\langle \frac{\Delta\tau_n^i}{\tau_f} \right\rangle = 0$. To calculate the average value $\left\langle \frac{\Delta\tau_n}{\tau_f} \right\rangle$ for the TOE focus, the obtained sum for 118 domains must be averaged over the whole area of focus, which gives the value $\left\langle \frac{\Delta\tau_n}{\tau_f} \right\rangle \approx 0.5$. Next, using the data for the TOE presented in [9] on the value of seismic moment $M_0 = 1.6 \times 10^{21}$ J and energy $E_s = 1.7 \times 10^{16}$ J, which is radiated in seismic waves, and assuming the modulus of shift at $\mu = 4 \times 10^{10}$ Pa through expression (4), we find that the average discharged stresses $\Delta\tau_n$ for the

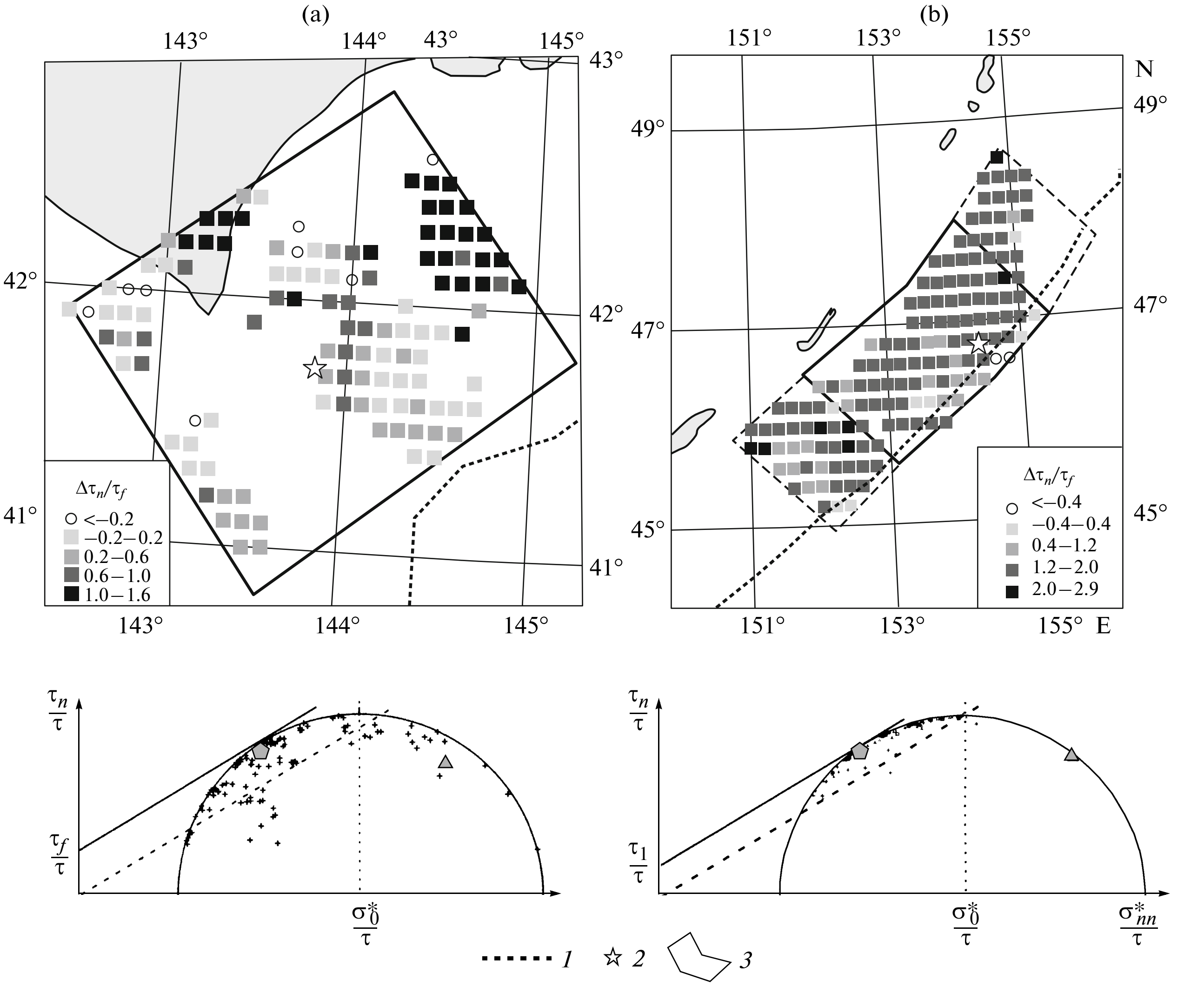

Fig. 2. Distribution of the values of discharged tangential stresses (valuation at adhesion τ_f, calculated within the framework of the third stage of the MCA algorithm for the TOE (a) and SE (b) events: (*1*) oceanic hollow, (*2*) beginning of "ripping" the earthquake focus (SE and TOE), (*3*) focus geometry used in calculations, (*4*) geometry of the SE focus by the USGS data. For the generalizing Mohr diagrams in the lower part of the figure, there is the position of points with values of the valuated normal and tangential stresses along the foci of these earthquakes. Valuation is made at the value of maximal tangential stress corresponding to the domain with parameters of the stress tensor, which is the closest to the beginning of earthquake "ripping." The line of minimal dry friction is shown also for the stress state of this domain. The points of some domains lie below the line of minimal dry friction, which is evidence for the difference of their stressed states from the state close to the beginning of earthquake "ripping". The tangent is the limit of effective strength; the dashed line is the line of minimal dry friction; the pentagon and triangle with light gray fill correspond to the stresses along the realized and conjugated nodal planes of focal mechanisms in TOE and SE events.

TOE focus were about 0.9 MPa (9 bar). If we use for calculation the data on geometrical sizes of the focus and expression (3), then the discharged stresses $\Delta\tau_n$ were 0.6 MPa (with $\chi = 1.85$, i.e., the focus area is close to isometric). Using both values of the discharged stresses, through expression (2) we find that the value of effective (smoothing scale is 100–200 km) internal adhesion τ_f is 1.2–1.8 MPa.

Another example of estimation of the value of effective adhesion of rock massifs can be given, using the reconstruction results of stresses acting in the crust of the northwestern margin of the Pacific seismoactive area before the SE event on November 15, 2006 (M_w = 8.3). The reconstruction was implemented in mesh points of the 0.2° × 0.2° orthogonal grid for the depth of 20 km by the data on focal mechanisms from the world catalogue of Harvard University (http://neic.usgs.gov/neis/sopar). The fullest range of earthquake magnitudes from this catalogue for the studied part of the crust is $5.5 > M_w > 6.5$, and the distribution density of their foci allowed us to tell about the smoothing scale of stress parameters of 50–70 km. To estimate τ_f, the seismologic data on energetic parameters of the SE event were used. According to the data presented on the Internet-site of Harvard University, the value of energy released in seismic waves E_s and the seismic moment M_0 of the SE were correspondingly 7.4×10^{16} and 3.4×10^{21} J. According to seismological data, the SE event has the following focal mechanism: the first nodal plane, strike 215°, sinking 15°; the second nodal plane, strike 33°, sinking 75°. The results of seismic record inversion [10] showed that it is the first nodal plane realized as the earthquake focus, and was corresponding to gentle sinking (15°) of the subducted oceanic plate under the subcontinental plate and had a length along the hollow (W) of about 400 km, while in the transverse direction (L) of about 130 km (dotted-line contour in Fig. 2b). The analysis of aftershock consequence distribution, made in [5, 7], gives correspondingly 300 and 60 km (solid-line contour in Fig. 2b).

Using the data on energy released during the SE, through expression (4) we find $\Delta\tau_n \approx 1.7$ MPa (17 bar). If we use expression (3) and data on the focus geometry (with $\chi = 1$, because the focal area is not isometric), then we will obtain $\Delta\tau_n \approx 0.5$ MPa (by the USGS data), $\Delta\tau_n \approx 1.4$ MPa (by [5, 11]). The presented results show that calculation of $\Delta\tau_n$ by the USGS data on the focal size understates the value, while the estimation of focal geometry in [5, 11] gives a better correspondence to the energetic parameters of the earthquake. In the following calculations, we will use the focus geometry given in [5, 11]. The value of discharged stresses in the SE focus can be obtained, with a precision to valuation at the unknown value of effective adhesion of rocks massifs τ_f, using the results of the first two stages of MCA reconstruction and expression (1). For the SE focus area, 90 domains exist for which the data on parameters of the stress tensor were obtained (Fig. 2b). Using the data of these domains and summing the contribution in discharged stresses for the parts of the focus within each domain, on the basis of expression (1) and data on the focal size, we find the average value of relation $\left\langle \frac{\Delta\tau_n}{\tau_f} \right\rangle = 1.41$. Next, using the data on values of discharged stresses, we find the value of effective adhesion $\tau_f = 1.0$–1.2 MPa, which is close to the estimation made above by the TOE data.

COMPARISON WITH DATA ON STRESS VALUES OBTAINED THROUGH OTHER METHODS

In MCA, the data on effective adhesion of rock massifs allow us to transit from relative values of deviator and isotropic stresses to their absolute ones. For the crust within the Kurile-Kamchatka and Japanese parts of the Pacific subduction zone, the range of changes in maximal tangential stresses is 2–40 MPa.

The known theoretical estimations of stress level τ at the depth of 30 km obtained in [12] are 0.7–1 GPa. In estimation of the stresses, these researchers rely upon the data on strength parameters obtained from the experiments performed on a specimen and used data on orientation of the main stresses corresponding to a certain geodynamical situation, as well as the hypothesis on proximity of vertical stress values to the weight of the overlying column of rocks and an assumption on the hydrostatic law of depth distribution of fluid pressure. The level of deviator stress equal to a few kilobars (hundreds of MPa) obtained through the estimation is hardly correlated with seismological data on the values of stresses discharged in foci of strong intraplate earthquakes, which are the first few bars or tens of bars. Such obviously overstated stresses are associated with the hypothesis on the hydrostatic law of depth distribution of fluid pressure and with the value of rock massifs adhesion, which is assumed at $\tau_f \approx 50$ MPa in these calculations. Note that this value of effective adhesion, according to Fig. 1, defines the minimal level of discharged stresses (if a focus plane corresponds to the plane of internal friction, and the displacement direction corresponds to the direction of tangential stresses acting on it before the earthquake).

Another result of estimation of the level of natural stresses is obtained in [13]. For her calculation, J. Hardebeck used the data on the orientation of the main stress axes before and after the strong Landers earthquake $M_b = 6.9$, which occurred in the San Andreas Fault in 1992, as well as the stress values discharged along the three different parts of the focus (of about 10 km length) of this earthquake ($5 \le \tau_n \le 10$ MPa). Hardebeck's method is oriented at a geodynamical regime typical for the investigated area of the shear stressed state and allows us to calculate the value of maximal tangential stress τ, which acted before the studied earthquake, on the assumption that the Lode–Nadai coefficient of stress tensor $\mu_\sigma = 0$. In the case of the Landers earthquake, it was established that τ changed along the future focus from south-southeast to north-northwest from 8 to 16 MPa. The effective internal friction, which can be estimated on the basis of [13], for different parts of the Landers earthquake changes from 1.7 to 3.7 MPa. The obtained estimations are close to the results presented in the current work and in previous ones [1].

CONCLUSIONS

Thus, the algorithm of the third stage of MCA presented in this paper allows us to estimate the effective strength of rock massifs with a smoothing scale of a few kilometers and more, and to define the values of tectonic stresses. The level of deviator stresses in the crust of the northwestern margin of the Pacific subduction area is 2–40 MPa, and the effective adhesion strength is about 1 MPa.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project nos. 06-05-64410 and 07-05-00106).

REFERENCES

1. Yu. L. Rebetskii (Nauka, Moscow, 2007) [in Russian].
2. J. Rais, *Earthquake Focus Mechanics, Ser. Mechanics. New in Foreign Science* (Mir, Moscow, 1982) [in Russian].
3. K. Kasahara, *Earthquake Mechanics* (Mir, Moscow, 1985) [in Russian].
4. B. V. Kostrov, *Mechanics of Tectonic Earthquake Focus* (Nauka, Moscow, 1975) [in Russian].
5. I. N. Tikhonov, N. F. Vasilenko, A. S. Prytkov, et al., in *Proceedings of International Scientific Symposium on the Problems of Seismic Safety of the Far East and Eastern Siberia* (IMGG DVO RAN, Yuzhno-Sakhalinsk, 2007).
6. http://earthquake.usgs.gov/eqcenter/eqinthenews/2006/usvcam
7. http://158.203.31.121/freesia/event/search/search.html
8. S. A. Fedotov, A. V. Solomatin, and S. D. Chernyshev, Vulkanol. Seismol., No. 5, 3–22 (2004).
9. http://earthquake.usgs.gov/eqcenter/eqinthenews/2003/uszdap
10. http://earthquake.usgs.gov/eqcenter/eqachives/poster/2006/20061115_image.php
11. S. A. Fedotov, A. V. Solomatin, and S. D. Chernyshev, Vulkanol. Seismol., No. 6, 3–23 (2008).
12. R. H. Sibson, Nature **249** (5457), 542–544 (1974).
13. J. L. Hardebeck and E. Hauksson, J. Geophys. Res. **106** (B10), 21859–21882 (2001).