Subordination of Microstructure Deformations and Brittle Macrodestructions

Yu. L. Rebetskii, R. A. Lementuyeva, N. I. D’yaur, and A. V. Mikhailova

Presented by Academician S.V. Gol’din December 24, 2004

Received December 29, 2004

The aim of our study was to distinguish the regularity of variations in the stress-and-strain state in the neighborhood of a brittle fracture at the stage preceding its appearance on the basis of laboratory experiments with rock sample breakdown. Cubic samples of limestone and sandstone, 10 x 10 cm in size, were loaded by injecting a nonexplosive destructive mixture (NDM) into a specially prepared cylindrical hole in the sample [1]. The measuring rosettes included three strain sensors pasted at an angle of \(45^\circ\) to each other on both horizontal and vertical planes of the sample at the exits of the tensile fracture formed during loading (Fig. 1c). Suplementing the strain sensor data on deformations \(\varepsilon_{xx}, \varepsilon_{yy},\) and \(\varepsilon_{xy}\) on the sample plane with expressions that determine the zero stress on lateral planes of the sample allowed us to calculate all six components of the strain tensor. We studied the regularities of variations in mean strain \(\theta,\) maximal shear strain \(\gamma,\) and in the values of the Lode–Nadai coefficient \(\mu_e,\) which determines the form of the strain tensor:

\[
\theta = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3}, \quad \gamma = \frac{\varepsilon_1 - \varepsilon_3}{2},
\]

\[
\mu_e = \frac{2(\varepsilon_2 - \varepsilon_3)}{\varepsilon_1 - \varepsilon_3} - 1.
\]

In Fig. 1, graphs of these parameters are shown in a dimensionless form on the time scale (the unity corresponds to the time of tensile fracture \(T_f\) appearance at the surface) and strain scale (normalization was performed by the value of maximal shear strain at the moment of fracturing). The initial stage of loading, which is accompanied by nonlinear effects related to nonlinear increases in the NDM load and the consolidation of the sandstone or limestone structure, was not used for the analysis.

We performed a preliminary theoretical analysis on the stressed state of samples for the elastic strain stage based on the results of analytical calculations and numerical modeling using the UWAY software [2]. The modeling demonstrated that coefficient \(\mu_e\) changes in the sample volume from a value equal to the ratio between internal external radiiuses (in our case, it was equal to 0.1, which corresponds to \(\varepsilon_3 = -0.1\)) near the internal surface of the pipe to \(-1\) at the external contour. Volume deformations are equal everywhere and depend only on internal pressure. The maximal shear strains linearly depend on internal pressure and change nonlinearly in the radial direction. Deformations of maximal contraction \(\varepsilon_3\) are formed in the pipe radius direction, while deformations of maximal extension \(\varepsilon_1\) are formed in the tangential direction.

Rosette 1, pasted on the horizontal surface of the sample near the NDM-containing inclusion, yielded \(\mu_e\) values close to \(-0.55\) at the initial elastic stage \((0.1T_f)\) (Fig. 1a). At this moment, intermediate main strains \(\varepsilon_2\) are formed normal to the lateral surface (axis \(z\)), while the strain components \(\theta\) and \(\gamma\) increase slowly and almost linearly. This stage terminates in a sharp drop of \(\mu_e\) to \(-1\), which corresponded to a reindexing of the main strain in the \(z\) axis direction \((\varepsilon_z = \varepsilon_3 = \varepsilon_{zz})\). After reindexing, \(\mu_e\) increased to \(+0.2\), which was accompanied by a sharp increase in the slope of the graph of strain components \(\theta\) and \(\gamma\). This stage had a duration of \(0.3T_f\) and terminated in the stabilization of the values of \(\mu_e, \theta,\) and \(\gamma\). It is possible that at this moment the energy of the external load was used for the formation of the tensile crack surface (fracture preparation stage). The short stabilization stage \((0.05T_f)\) gave way to a sharp drop of \(\mu_e\) to zero, accompanied by a new stage of rapid increase in the values of \(\theta\) and \(\gamma\) at the moment of macrofracture exhumation to the upper surface of the sample. The analysis of strain characteristics based on rosette 1 allowed us to distinguish three stages: (1) linear elastic deformation; (2) nonlinear deformation; and (3) preparation of the fracture and breakdown of the sample. Nonlinearity of deformation is related to the accumulation of defects in the sample interior near the cylindrical inclusion. In the course of loading, this zone
Gradually widened and involved zones of future tensile fracture in the nonlinear deformation stage.

Rosettes 2 and 3 were pasted on the vertical lateral surface of the sample (Figs. 1b, 1d). Since the hole in the sample for NDM filling was drilled to 4/5 of the depth, the mean $\mu_e$ values at the initial elastic stage were close to –0.6 for rosette 2, pasted in the upper part of the sample surface, and close to –0.3 for rosette 3, which was located near the bottom level of the NDM-containing cylindrical hole. In both cases, algebraically minimal deformations $\varepsilon_3$ are formed in the normal direction to the surface (axis $z$), and the increase in $\theta$ and $\gamma$ values at this stage is close to linear. During the nonlinear stage determined on the basis of the analysis of rosette 1 data, the $\mu_e$ values drop to –1. At these moments, the main strain is reindexed in the direction perpendicular to the vertical surface of the sample ($\varepsilon_3 = \varepsilon_3 = \varepsilon_{zz}$), after which intermediate main strains $\varepsilon_2$ are formed in this direction. At first, the reindexing occurs at rosette 3. As in rosette 1, the reindexing in rosette 2 after some delay is caused by the nonlinearity of the strain developing from the NDM-containing cylindrical inclusion in the sample interior.

The reindexing is followed by an increase in the slope of $\theta$ and $\gamma$ graphs (except for $\theta$ at rosette 3), a rapid increase in $\mu_e$ to –0.5, and the commencement of the stabilization stage. It is likely that the nonlinearity in the strain parameter graphs for rosettes 2 and 3 of this stage is caused by the development of the nonlinear deformation segment near the inclusion. The stabilization stage $\mu_e$ is followed by a new increase in the slope of $\theta$ and $\gamma$ graphs. It is possible that the increase in

Fig. 1. (a, b, d) Variation of characteristics $\theta$, $\gamma$, and $\mu_e$ in the sample based on the data from rosettes 1, 2, and 3 (c) Schematic location of the rosettes, position of the cylindrical inclusion with NDM, and the exhumation of tensile fracture on the horizontal surface of the sample.
strains is related to the appearance of a tensile fracture in the sample interior near the NDM-containing cylindrical inclusion and to the expansion of zones involved in nonlinear deformation up to the vertical surface. In this case, the $\mu_e$ value increases to $-0.2$ for rosette 2 and decreases to $-0.8$ for rosette 3. The last stage of increase in the slope of $\theta$ and $\gamma$ graphs is related to the exhuma-

tion of the tensile fracture. Thus, the analysis of strain characteristics based on rosettes 2 and 3 allowed us to
distinguish four stages: (1) linear elastic deformation; (2) nonlinear deformation near the inclusion; (3) the
appearance of the tensile fracture near the inclusion and nonlinear deformation near the vertical surface; and
(4) preparation of the fracture and its appearance near the vertical surface of the sample.

Taken together, the analytical and numerical modeling results correspond well to the data obtained in the
experiments using strain sensors at the initial elastic stage. At the same time, UWAY-based calculations did
not yield any variations in the parameters of the stress-
and-strain state such as were observed in the experi-
ments at the brittle fracture stage (we considered a lin-
early elastic model with the tensile fracture growing
from one of the planes). The experimental result may be
related to the following fact: samples subjected to brit-
tle deformation are marked by a stage in which defects accumulate in the nonlinear deformation zone, and this
stage was absent in the numerical modeling.

Indirect data on the nature of the process at the non-
linear deformation stage were also obtained on the
basis of electric potential (EP) and ultrasound profiling
(USP) data [1]. It was found that this stage is character-
ized by an increase in the EP growth rate, and its varia-
tion mode is similar to that of variations in the mean
deformation $\theta$. Initially, the EP growth rate is small, but
later it begins to increase sharply. One can see a certain
advance in the EP variation relative to the $\theta$ variation.
The USP-based elastic wave velocities remain virtually
constant for a long period (the EP value at this stage is
already 50% higher relative to the linear deformation
stage) and they sharply decrease only immediately
prior to the appearance of a macrofracture.

Comprehensive analysis of the results of our exper-
iments and data reported in [3] allowed us to make a
supposition concerning the peculiarities of the micro-
structural transformations that precede brittle destruc-
tion and that are reflected as variations in strain charac-
teristics. The microstructural transformations, which
occur in very narrow zones at the nonlinear deformation
stage, most likely generate an echelon of microshears and subordinate microfractures. They are ori-
ented in accordance with the Coulomb theory of
strength and lead to the partial removal of elastic defor-
mations in the direction of two main (algebraically
maximal and minimal) strain axes, which are reple-
ished owing to the continued loading. In turn, micro-
fractures accompanying the appearance of microshears
decrease the effective tensile strength. On the whole,

microstructural transformations can be defined as plas-
tic deformations, the appearance and development of
which decrease the strength. Such microstructural
transformations lead to a decrease in the energy spent
on destruction.

It is noteworthy that the performed experiments
were characterized by some peculiarities, owing to
which it is not possible to estimate their results from the
standpoint of the classic mechanics of brittle destruction.
In particular, the loading rate appeared to be small
relative to the microdestruction rate. Therefore, the
plastic deformation zone is more extended than the
characteristic dimensions of the model. Thus, the prop-
erties of the microfracture formation zone differ from
those of the environment. Therefore, one cannot say
that the growth rate of the deformation zone is related
only to the coefficient of stresses in the crack mouth.
Under such conditions, the growth rate of macrofrac-
tures and the critical parameters of growth stages will
be related not only to the loading mechanism, but also
to the rate of structural transformations. At certain
moments of time, the energy accumulated in micro-
structures (internal stresses) can be sufficient for the
growth of a macrofracture even after the termination of
further loading.

Let us analyze some possible variations in the defor-
mation regime of the future brittle fracture zone at the
microstructural transformation stage for a point on the
sample surface near the inclusion (comparison with the
data on rosette 1). This zone is marked by plastic
unloading owing to shear displacements along the Cou-
lomb–Moore microfractures, which are formed along
the shear planes. The development of microdisplace-
ments leads to a decrease in the level of maximal shear
stress in the radial direction by $\Delta \sigma$. Owing to a decrease
in the intensity of radial compression (given that the
level of other main stresses is retained), elastic defor-
mations are redistributed along the axes of the main
stresses. If we consider that the orientation of the main
axes of stress does not change at these transformations
(the cleavage plane is close to the plane of maximal tan-
gent stresses $\tau_{13}$), then the increments of elastic defor-
mations in the maximal shear deformation will be equal
to the increments of the residual (plastic) shear defor-
mations. It is possible to show that in this case the values
of the parameters under study would be determined
by the following relations:

\begin{equation}
\mu_e = \mu_e^0 - \kappa \frac{\Delta \sigma}{\gamma},
\end{equation}

\begin{equation}
\theta = \theta^0 + \lambda \Delta \sigma, \quad \gamma = \gamma^0,
\end{equation}

where $\kappa$ and $\lambda$ are elastic constants depending on the
elastic modulus and Poisson coefficient, and the upper
index equal to zero denotes the value of the respective
parameters before the appearance of a microshear
defformation. As seen from relations (2), the value of
coefficient $\mu_e$ at the microstructural transformation
SUBORDINATION OF MICROSTRUCTURE DEFORMATIONS

The stage should be close to \(-1\) (uniaxial elongation). Reindexing of the axes of main deformations occurs when the deformed state corresponds to uniaxial elongation. At this point, deformations \(\varepsilon_z\) are formed at the analyzed macropoint in the direction \(z\) perpendicular to the sample surface.

After equalizing the main elastic deformations with indices 2 and 3, further removal of elastic deformations may occur due to both displacements along microshears in the planes perpendicular to the horizontal surface and displacements along new microshears, whose planes are inclined at an angle of \(45^\circ\) to the \(z\) axis and tangential direction (plane of action \(\tau_{13}\) before reindexing). In this case, the form of the stress tensor should always remain close to uniaxial extension (\(\mu_{\sigma} \approx -1\)). However, this is not observed in the experiment. Here, strains \(\varepsilon_z\) are formed in the \(z\) axis direction after reindexing and up to the point of destruction. This situation is possible if we suppose that microshears formed at the previous stage changed strength properties of the medium. The old microshears lock the zone and hamper the formation of microshears in other directions (anisotropy of strength). After reindexing of the axes of main deformations, the accumulation of residual strains and release of elastic energy continues owing to microshears formed in the plane of tangential stresses \(\tau_{12}\) (plane \(\tau_{13}\) before reindexing). This stage is marked by an increase in the maximal tangential stress \(\tau_{13}\) up to the critical value. This is followed by an avalanche formation of microshears and hierarchic microfractures in the plane of tangential stress \(\tau_{13}\) action. At this stage, zones of higher dilatancy and lower strength are developed in the structural rearrangement zone owing to the formation of two conjugated systems of fractures and cleavages. The widening and merging of such zones is the last occurrence in the microstructural rearrangement before the formation of tensile macrofracture.

The results of our experiments make it possible to explain the nature of regular variations in the tectonic stresses revealed by tectonophysical reconstruction. For example, a reconstruction of the modern stressed state of the Kuril–Kamchatka seismoactive zone using data on earthquake focus mechanisms [4] allowed us to distinguish periodic variations accompanied by a sign change in the Lode–Nadai coefficient. Strong earthquakes (\(M_b > 7.5\)) are always preceded by a decreased \(\mu_{\sigma}\) stage, whose duration is related to the magnitude of the event, whereas the earthquake itself occurs at the stage of \(\mu_{\sigma}\) growth. Reconstruction of paleostresses based on the shear fracture data [5] at magmatic, hydrothermal, and hydrothermal-metasomatic deposits demonstrated that ore bodies are usually confined to rock zones that are characterized by a high variability in the orientation of the main stress axes and by the widest range of \(\mu_{\sigma}\) variations.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research, projects nos. 03-05-64709, 03-05-64998, and 03-05-65092.

REFERENCES