Doklady Earth Sciences, Vol. 365A, No. 3, 1999, pp. 370–373. Translated from Doklady Akademii Nauk, Vol. 365, No. 3, 1999, pp. 392–395.

GEOPHYSICS

Methods for Reconstructing Tectonic Stresses and Seismotectonic Deformations Based on the Modern Theory of Plasticity

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Presented by Academician V.N. Strakhov July 11, 1997

Received July 30, 1997

The main problem of reconstructing tectonic stress tensors and the rate of seismotectonic deformation by shear fractures is the creation of homogeneous samples of initial structural kinematic data (SKD) defining the boundaries of homogeneously deformed domains in, generally speaking, 4D volume with the coordinates of space and time. Following [1, 2], we offer the reconstruction method based on the principles of the plastic flow theory and strengthening extended for the description of deformation of fractured rock massifs. The process of irreversible strain accumulation, due to the sliding of a multitude of random-oriented crosscutting faults in rock massifs along the surfaces of reduced strength, is referred to as a cataclastic flow, and the methods of analyzing the regularities of this flow are the methods of cataclastic analysis.

We shall consider Mises's maximum principle, which is fundamental for the construction of the modern theory of plasticity

$$(\sigma_{ij} - \sigma_{ij}^*) de_{ij}^p \geq 0, \tag{1}$$

defining the convexity of loading surface Σ (Fig. 1), constructed in the 6D space of stresses [3]. In (1), σ_{ij} are the actual values of stress components corresponding to the given value de_{ij}^p, and σ_{ij}^* are the components of any possible stressed condition inside the loading surface. From (1) and Fig. 1, it is seen that the vector of increments of plastic deformations de^p is associated with Σ. Thus, in all cases except for plastic unloading, the product of tensors of plastic strain and stress increments cannot be negative,

$$d\sigma_{ij} de_{ij}^p \geq 0, \tag{2}$$

and the creation of irreversible deformation requires the additional work

$$\sigma_{ij} de_{ij}^p \geq 0. \tag{3}$$

The validity of (2) determines the stage of a stable elastoplastic deformation (active loading). In Fig. 1, an acute angle between the respective vectors correspond to inequalities (2), (3).

Consequently, the condition of ordering stress tensor components and plastic deformation rates [4] is taken from (1)

$$(\sigma_i - \sigma_j)(de_{ii}^p - de_{jj}^p) \geq 0, \quad i, j = 1, 2, 3. \tag{4}$$

Here, de_{ii}^p are components of the tensor of plastic elongation and shortening deformation increments along

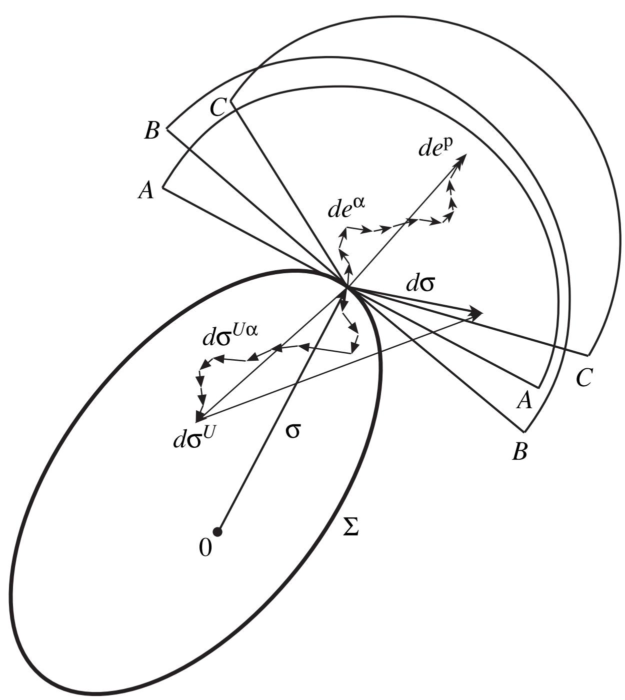

Fig. 1. A surface of elastoplastic loading Σ in 6D stress space.

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main stress axes σ_i. Choosing indexes in (4) so that the difference between the main stresses was positive, we discover that

$$de_{11}^p \geq de_{22}^p \geq de_{33}^p; \quad (5)$$

i.e., in operation directions of main stress axes, the irreversible plastic elongation and shortening deformations are formed in correspondence with their indices. The last expression imposes a restriction on the plastic flow character for a wide range of materials, including anisotropic mediums.

In the description of cataclastic deformation, we understand σ_{ij} as the effective stresses average for homogeneous domains and de_{ij}^p as the respective increments of residual (fracture-confined) strains. Considering each displacement along the shear fracture (seismic center) as an outcome of plastic deformation microact and following the work [5],

$$de_{ij}^\alpha = \frac{\Omega^\alpha D^\alpha}{2V^\alpha}(n_i^\alpha s_j^\alpha + n_j^\alpha s_i^\alpha), \quad i, j = 1, 2, 3. \quad (6)$$

Here, tensor de_{ij}^α determines the average (for all points of elastic unloading region V^α) contribution to the tensor of seismotectonic fracture of α, while n_i^α and s_i^α are the guiding cosines of vectors of the normal to the fracture plane and displacements along the latter, respectively (i.e., the nodal plane normals for seismological data on earthquake center mechanisms), in an arbitrary coordinate system; D^α is the average displacement along the fracture plane; and Ω^α is the fracture area. The standard method for calculating the tensor of seismotectonic deformation increments (rate) is known [6], which is based on the summation of separate contributions to the process of residual strain accumulation, in accordance with (6), of different earthquakes the seismic centers of which enter the arbitrarily defined macrovolume of averaging. In [6], it is shown that within the framework of the approach presented in this article, the calculation of the tensor of seismotectonic deformation increments is to be executed on the basis of expression

$$S_{ij} = d\gamma \sum_{\alpha=1}^{A} (n_i^\alpha s_j^\alpha + n_j^\alpha s_i^\alpha)/2, \quad i, j = 1, 2, 3, \quad (7)$$

where $d\gamma$ determines the average (for the regions of elastic unloading V^α) value of the maximum elastic shear strain measured after the earthquake. The quantity $d\gamma$ characterizes the plastic flow ability of the respective rock massifs and is assumed close to constant for a homogeneous SKD sample. The first criterion of the SKD sample homogeneity and the possibility to calculate S_{ij} by means of expression (7) is the availability of a crosscutting zone of the regions of elastic unloading of earthquakes. The value of the calcu-

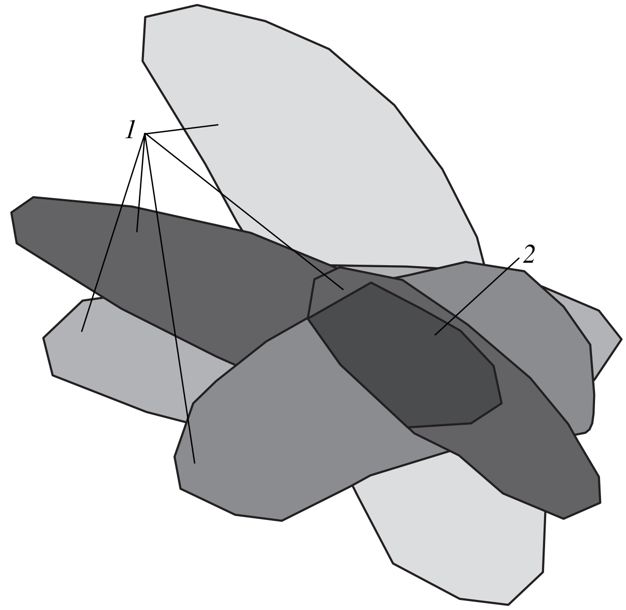

Fig. 2. A zone of (*2*) mutual intersection of (*1*) regions of elastic unloading for a set of earthquakes.

lated tensor of seismotectonic deformations increments (Fig. 2) is assigned to respective points of the zone. Note that (7) coincides accurate within normalization with the known expression for calculating the average mechanism according to Yunga [7].

In [1, 2], it is shown that the use of the principle of dissipation positiveness over the true tensor of stresses (3) for each of the shear faults of the homogeneous SKD sample results in the following restriction on orientation of sliding along the shear surface for an arbitrarily oriented fault plane:

$$n_1^\alpha s_1^\alpha \geq 0, \quad n_3^\alpha s_3^\alpha \leq 0. \quad (8)$$

Due to the similarity of expressions (2) and (3), the last inequalities define the condition of active loading of a homogeneous domain in the process of cataclastic deformation. The inequalities (8), however, obtained on the basis of the dislocation plasticity theory, are necessary for creation of homogeneous SKD samples within the framework of reconstruction of stresses according to the “right dihedra” [8, 9] and “kinematic” methods [10]. Thus, contrary to the statements expressed in [11], their correctness is confirmed from the point of view of validity of mechanical energy dissipation positiveness.

The existence of an ordered state of elastoplastic flow (4) during its propagation over each microact of plastic deformation allows us to write the following criterion of creating homogeneous SKD samples from (5) on the basis of (6):

$$n_1^\alpha s_1^\alpha \geq n_2^\alpha s_2^\alpha \geq n_3^\alpha s_3^\alpha. \quad (9)$$

Given that (9) is valid and vectors $\mathbf{n}^\alpha$ and $\mathbf{s}^\alpha$ are collinear, formula (8) is obtained automatically. In the

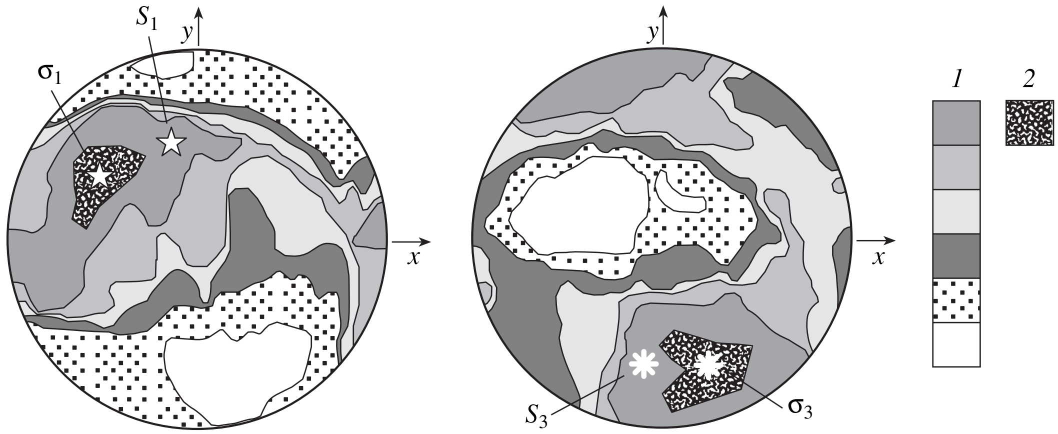

Fig. 3. Regions of allowable orientation of (*1*) increments of seismotectonic deformations and (*2*) principal axes of stresses, selected on spheres of a unit radius on the basis of limiting inequalities, for a set of shears in a homogeneous SKD sample and emergences of principal axes of corresponding tensors.

stress space illustrated in Fig. 1, the realization of inequalities (8) and (9) can be presented as sectors *AA* and *CC*, respectively, within the limits of which vectors $\mathbf{de}^{\alpha}$ corresponding to the required effective tectonic stress tensor σ_{ij} must be placed. Thus, the parametrical region, within the limits of which the inequalities (9) are valid, is also the region of allowable solutions for the tensor of sought tectonic stresses (Fig. 3) in the construction of these regions on a sphere of single radius, as in the case of the "right dihedra" method (the latter regions create restrictions on the spectrum of probable orientations of principal stress axes). The criterion of existence of the region satisfying the restrictions of (9) for a set of earthquakes is the second criterion for creating a homogeneous SKD sample.

Further, note that in the case of plane fractures, the stresses measured in the field of elastic unloading which resulted by shear displacement, comply with the condition of pure shift, and the vector of measured stresses $\mathbf{d\sigma}^{U\alpha}(d\sigma_{ij}^{U\alpha} = -G_{ns}^{\alpha}de_{ij}^{\alpha}$, where G_{ns}^{α} is the module of the shift in a plane formed by vectors $\mathbf{n}^{\alpha}$ and $\mathbf{s}^{\alpha}$) should be directed inward towards the loading surface (Fig. 1)

$$d\sigma_{ij}^{U\alpha}de_{ij}^{p} = -G_{ns}^{\alpha}de_{ij}^{\alpha}de_{ij}^{p} \le 0. \quad (10)$$

Assume that for a steadily developing deforming process the tensor of seismotectonic deformation increments tends to be similar to the full plastic deformations tensor $de_{ij}^{p} = kS_{ij}$ ($de_{i}^{p} = 0$, $k = \text{const} > 0$). Then, using expression (6) on the basis of (10), written against the principal axis of the tensor of seismotectonic deformation increments, we obtain

$$n_i^{\alpha}s_i^{\alpha}S_i \ge 0. \quad (11)$$

Restriction (11) for an arbitrary and previously unknown value of the coefficient of the kind of seismotectonic deformation increment tensor is valid on condition that inequalities such as (8) are valid. Thus, in expression (8), n_i^{α} and s_i^{α} will represent the guiding cosines in a system of principal axes of the seismotectonic deformation increment tensor. In the space of stresses shown in Fig. 1, the operation of these inequalities can be represented as sectors *BB*, which should accommodate vectors $\mathbf{de}^{\alpha}$ that are adequate to the sought seismotectonic deformation increment tensor S_{ij}. Thus, the parametrical region, within the limits of which the inequalities (8) hold, is the region of allowable solutions of the tensor of calculable seismotectonic deformation increments. The criterion of existence of the region, which satisfies restrictions (8) for a set of earthquakes, is the third criterion for creating a homogeneous SKD sample.

Thus, within the framework of the common approach based on the principles of cataclastic analysis, the criteria for creating homogeneous SKD samples for shear fractures are formulated, making it possible to calculate the tensor of seismotectonic deformation increments and determine regions of probable orientations of principal axes of the sought tectonic stress tensor. The true parameters of stresses in a context of the proposed method should be evaluated on the basis of requirements of maximum mechanical energy dissipation [11]. For this purpose, on a sphere of unit radius in the regions of allowable, according to inequalities (9), orientations of the axes of principal stresses, one should find such a position of these axes and determine such a value of Lode–Nadai coefficient under which the Mises maximum condition for the calculated tensor of seismotectonic deformation increments is satisfied.

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