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Retrieval of Principal Stresses in Crust from Their Trajectory Field

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It is proposed that a method known from simulation with optically active materials and from plasticity theory (by a method based on integration along the trajectories of the principal normal stresses) be used in solving the problem of retrieval of the values of the principal stresses operative in the crust. A modification of the method applicable to tectonophysical problems is given which is based on use of data obtained by the O. I. Gushchenko kinematic method, supplying information on the trajectories of the principal stresses and the value of the type coefficient of the stress state.

In tectonophysics research in the final analysis the problem of determining the deformation mechanism is posed, or more narrowly, the method for loading of a particular region. Since the investigated body, generally speaking, is inhomogeneous, has anisotropic properties and in the deformation process experiences great deformations, a study of the stress field will be most promising in the solution of this problem. It is precisely the stress fields which link deformation processes of sectors of the crust which are different in their properties into a single system and therefore precisely on the basis of a study of the nature of the distribution of stresses it is possible to draw conclusions on the general mechanism of deformation in a region. We will demonstrate that the attained level of tectonophysics research makes it possible to formulate the problem of retrieving the magnitude of the principal stresses operative in the crust.

A number of studies devoted to development of methods for retrieving the trajectories of the principal normal stresses in the crust have recently appeared [1-9]. The basis for these methods is an analysis of fault dislocations, data on which are supplied by the field tectonophysics method, as well as by seismological methods. All these studies in one way or another have their roots in the work of M. V. Gzovskiy [1], the first to define this tectonophysics problem. If these methods are concisely classified, they can be divided into two groups. The first includes methods based on the assumption that the rupture surfaces either coincide with the plane of operation of the maximum shear stress or deviate from it by the value of shear angle [1-5]. There is also another group of methods based on broader concepts [6-9]. It is assumed that movements occur, including along fault dislocations already existing in the medium, but the direction of movement along them coincides with the direction of the shear stresses operating here.

The approach proposed in this study will be based on data obtained using the O. I. Gushchenko method [7], whose use has now made it possible to ascertain the fields of trajectories of the principal normal stresses, regularly changing in space. It is known that this method, in addition to the orientation of the principal stresses at discrete points in the medium (the values of the direction cosines of the vectors of the principal normal stresses), also makes it possible to determine the values of the Lode-Nadai coefficient (coefficient of the type of stress state)

$$M_\sigma = 2\,\frac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3} - 1, \tag{1}$$

where σ_i ($i = 1, 2, 3$) are the principal normal stresses. By knowing M_σ, at each point it is possible to find the ratio of stress deviators s_i ($s_1 + s_2 + s_3 = 0$):

$$\frac{s_2}{s_1} = \frac{2M_\sigma}{3-M_\sigma}, \quad \frac{s_3}{s_1} = -\frac{3-M_\sigma}{3+M_\sigma}, \tag{2}$$

where $s_i = \sigma_i - \sigma_0$, where σ_0 is spherical stress. Since the value of the stress deviator s_1 by which terms in (2) are reduced to dimensionless form is, generally speaking, unknown and varies from point to point, it is impossible to compare the dimensionless stress deviators at different spatial points obtained in this way.

We will formulate the problem to whose solution this article will be devoted. Assume that for some sector of the crust adjacent to the surface the field of trajectories of the principal normal stresses σ_i, or, in other words, the field of direction cosines, is known, as well as the field of M_σ values. It is necessary to retrieve the values of the principal normal stresses σ_i.

We will write the equilibrium equations in a curvilinear orthogonal coordinate system [10] coinciding with the trajectories of the principal normal stresses:

$$\sigma_{i,i} + \sum_{j=1}^{3} H_{j,i}(\sigma_i - \sigma_j)/H_j + H_i F_i = 0, \quad i = 1, 2, 3. \tag{3}$$

Here F_i is the projection of the specific volumetric force onto the coordinate lines i, H_j are the Lamé coefficients [11], and $H_{j,i}$, $\sigma_{i,i}$ are derivatives of i. Hereafter we will assume that the density distribution in the investigated region is known.

In the selected coordinate system we have a system of three first-degree differential equations in partial derivatives with three unknown $\sigma_{i,i}$ functions. It is known that the classical writing of the equilibrium equations in a Cartesian system of orthogonal coordinates has the form

$$\sum_{k=1}^{3} \sigma_{rk,k} + F_r = 0, \quad r = 1, 2, 3, \tag{4}$$

where σ_{rk} are the six stress tensor components.

In the classical approach for the solution of system (4) it is supplemented by expressions relating stresses to the derivatives of movements. As a result the solving system is a system of nine differential equations with six unknown components of the stress tensor and three unknown components of the vector of movements, which include rheological constants determining medium properties. In our case, however, as a result of conversion to a curvilinear orthogonal coordinate system, based on the trajectories of the principal stresses, on the one hand it is possible to reduce the degree of the resolvents, and on the other hand, to exclude the rheological constants unknown for rock masses. The decrease in the number of unknowns in the equilibrium equations is related to the fact that on orthogonal surfaces of the selected coordinate system the shear stresses are equal to zero. In the final analysis the simplification of Eqs. (3) is a result of use of a part of the information obtained by the kinematic method: data on orientation of the principal normal stresses.

In order to solve system (3) it is necessary to formulate the boundary value problem, that is, it is necessary to stipulate the conditions on three boundary surfaces (the differential equations entering into the system are of the first degree). In stipulating the form of the outer boundaries of the investigated sector of the crust we will neglect the general curvature of the surface, determined by the figure of the geoid, and relief irregularities. In this case it is possible to formulate the problem for a half-space on whose surface there is no tangential load, whereas the normal load is determined by the deviation of relief from the mean level. Thus, the surface of the half-space is the surface of operation of the principal normal stress. The boundary condition, stipulated at the surface, is only one of the three boundary conditions necessary for solving the differential system of Eqs. (3). If the investigated sector of the crust does not have other natural boundaries where it would be possible to stipulate boundary conditions on the basis of one concept or another, as the two other boundaries of the volume it is possible to use the surface of operation of the principal normal stresses.

Since in actuality it is virtually impossible to formulate boundary conditions in stresses for three internal boundaries, in stipulating the boundary conditions here we will use still other information supplied by the kinematic method together with the trajectories, the value of the Lode-Nadai coefficient, that is, we will stipulate the ratio of the principal stresses (1) as the known quantity at these boundaries.

It must be noted that since the M_σ value is known in the entire investigated volume, it must be consistent with the trajectories of the principal normal stresses. The degree of this consistency, dependent on the accuracy in determining M_σ and the accuracy in retrieving trajectories by the kinematic method, in the final analysis also determines the accuracy in retrieving the principal stresses. The requirement of consistency of the trajectory field with the M_σ field can be represented in the form of a differential equation derived by means of excluding σ_i from system from system (1), (3). The "consistency" equation derived in such a way, which includes the partial derivatives of H_i, F_i, M_σ relates the geometry of the field of trajectories of the principal stresses and the field of the Lode-Nadai coefficient and the nature of the density distribution. A solution of system (1), (3) exists only in the case of satisfaction of the "consistency" equation. Thus, the "consistency" equation of system (1), (3) is a test equation for the initial information, that is, for H_i, F_i, M_σ, only after whose identical satisfaction is it possible to proceed to the solution of system (1), (3).

The formulated approach to solution of the outlined problem must be supplemented by an examination of the problem of the uniqueness of the solution of system (1), (3). However, the problem of proof of uniqueness is a separate, quite complex problem and will not be examined in this article.

The stipulation of the boundary conditions of system (3) in stresses makes it possible to reduce the problem to a class of statically determinable problems in mechanics, that is, to problems in retrieval of a stress state in a deformed volume, in the solution of which there is no need for making use of equations relating stresses and strains, and accordingly, no need for a knowledge of properties of the medium. In actuality, the properties of the medium are already embodied in the nature of the trajectory field, as well as in the value of the coefficient M_σ, which we obtained on the basis of a method not directly related to solution of the equations of mechanics.

Methods for retrieving the principal stresses from their trajectories, similar to that proposed, are used in the polarization-optical simulation of a plane stress state [12], and also in plasticity theory [13]. Here the possibility of computing stresses is related to the choice of medium models: elastic in the case of optical simulation and ideally plastic in plasticity theory.

In our problem the possibility of computing stresses is unrelated to the choice of any specific model of the medium because information on the trajectory field and the M_σ coefficient was obtained on the basis of a method which imposes no limitations on the medium.

In addition to an approach based on the integration of equations (3), another approach also can be suggested. It is related to solution of the equilibrium equations written in a Cartesian coordinate system (4). Using data on the direction cosines of the principal normal stresses, we supplement (4) by expressions relating the principal stresses σ_i and stresses on surfaces the normals to which are the axes of the Cartesian coordinate system, and we obtain

$$\sigma_{rk} = \sum_{i=1}^{3} \sigma_i l_{ir} l_{ik}, \quad r, k = 1, 2, 3. \qquad (5)$$

Here l_{ir} are the direction cosines between the coordinate axes i (principal stress axes) and r (axes of a Cartesian coordinate system, one of which is vertical to the crustal surface). Substituting (5) into (4) and differentiating, we obtain:

$$\sum_{k=1}^{3} \sum_{i=1}^{3} (l_{ir} l_{ik} \sigma_{i,k} + \sigma_i (l_{ir} l_{ik,k} + l_{ik} l_{ir,k})) + F_r = 0, \quad r = 1, 2, 3. \qquad (6)$$

Thus, as in the case of system (3), we have a system of three first-degree differential equations in partial derivatives, but in contrast to (3) it is written in Cartesian coordinates, and what is most important, the coefficients in it are not the geometrical characteristics of the trajectories, but the direction cosines of the principal stresses. The latter circumstance is significant because the main difficulties and errors in solution of system (3) are related to the accuracy in retrieving the trajectories in space, the accuracy in finding the points of their intersection. It is necessary to note a better coupling between the equations of system (6) than is true for the equations of system (3).

It is evident that a direct reduction of the system of differential equations to systems of linear algebraic equations is possible in both variants of writing of the resolving system for the problem of stress retrieval. For example, if the partial derivatives are represented in finite differences, the systems of differential equations (3), (6) can be reduced to solution of a system of linear algebraic equations

$$\sum_{m=1}^{L} A_{tm} X_m = C_t, \quad t = 1, 2, \ldots, L. \qquad (7)$$

Here $L = 3N$, N is the number of points of intersection in the finite-difference grid and $\sigma_i = X_p$ where $p = (i-1)N + n$ $(n = 1, 2, \ldots, N)$. The first $L-(S_1 + S_2 + S_3)$ equations of system (10) represent equations (3) or (6), written in finite differences; the next $(S_1 + S_2 + S_3)$ equations represent the boundary conditions at the surface in stresses and the conditions at the other two lateral surfaces, stipulated by the relation of the principal stresses (1).

In the proposed approaches to solution of systems (3), (6) the part of the information supplied by the kinematic method, that is, the known values of the M_σ coefficient within the investigated volume, remains unused. However, since the trajectories of the principal stresses and the M_σ coefficient are determined from field data with a definite degree of accuracy, in order to increase the accuracy in retrieving the principal stresses it would already be necessary when solving system (3) or (6) to use all the information on the nature of the stress field. For this we supplement the system of Eqs. (3) or (6) by the relations for the principal stresses (1) within the investigated region. In this case the system becomes overdetermined: there are more equations than there are unknowns. As already mentioned above, there is no error here since in actuality if the trajectories of the principal stresses and the values of the Lode-Nadai coefficient were determined precisely, some of these equations would satisfy identity. Such an overdetermination of the system of equations makes it possible to proceed to the finding of an optimal solution based on the methods employed in solving incorrect problems. On the basis of the expressions for M_σ (1) we write a system of differential Eqs. (3) or (6), represented in finite difference form (7), and also from S_0 known values of the principal normal stresses (this includes both points belonging to the surface and points within the region, the values of stresses at which are known) we find a quadratic functional, by minimizing which for σ_i we obtain:

$$\sum_{m=1}^{L} \Big[\sum_{s=1}^{K} [A_{ts} A_{ms}] X_m \Big] - \sum_{s=1}^{K} A_{ts} C_s = 0, \quad t = 1, 2, \ldots, L. \qquad (8)$$

where $K = 4N + S_0$. The use of a modified method for solving the system of differential equations, that is, the reduction of (1), (3) or (1), (6) to (8), makes it possible to adjust the computation errors caused by the inaccuracy in determining the orientation of the vectors of the principal stresses and the values of the Lode-Nadai coefficient.

Conclusions. 1. An approach to solution of the problem of determining the values of the principal normal stresses operative in the crust is formulated which is based on information supplied by the kinematic method. 2. A method is proposed for writing a consistency equation for checking the initial information for reciprocal consistency. 3. A solving system of equations is written for determining the values of the principal stresses, taking into account the entire volume of information supplied by the kinematic method.

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